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THE CHALLENGE OF THE INITIAL TRAINING OF MATHEMATICS TEACHERS: KNOWLEDGE, PRACTICE, AND IDENTITY

Mª Isabel Pascual, & Juan Pedro Martín
Research center COIDESO, University of Huelva (Spain)

Abstract

In this research we pretend to answer the question: how is the initial training of mathematics teachers in the field of geometry? We argue that it is especially relevant to examine the content of mathematics teacher training in order to improve training processes. For this, we have analyzed training tasks and their management by the teacher educator about polygons, the construction of the definition and different classifications. The results show the variety of knowledge that is combined in this stage, including professional knowledge, professional practices and professional identity, providing evidence of theoretical advances in the field.

Keywords: *Initial teacher training, didactics of geometry, professional knowledge, professional practices, professional identity.*

1. Introduction

To answer the questions "What do prospective primary teachers (PPTs) need to learn to teach mathematics?" and "How do they learn it?" is mediated by the vision of what mathematics is for those who answer it, as teachers and as teacher educators. Also, it has to do with our beliefs about what a teacher is, beyond their knowledge. Initial teacher training is, therefore, heterogeneous in terms of its structure and content (Watson & Mason, 2007; Sánchez & García, 2008).

During initial training, the PPTs must make connections between the theoretical ideas that emerge from the field of Mathematics Education —theories, models, difficulties around a topic— and the implementation of these ideas in designs and planning (Jaworski and Huang, 2014). Likewise, in line with Ponte (2012), we consider that the content of initial teacher training is articulated around three main axes: professional knowledge, teaching practices and professional identity.

In the Spanish university context, the PPTs receive university degree training for four years, in which the courses aimed at developing their knowledge about the teaching and learning of mathematics is variable. Sometimes, some study programmes divide the subjects according to whether they work mainly on mathematical knowledge (MK) or pedagogical content knowledge (PCK), and on others, the subjects integrate both knowledge around the main mathematical topics (Nolla et al., 2021). According to the context in which this research has been developed, we are going to consider the second of the training models.

To explore the content addressed in the initial mathematics teacher training classroom, we are going to use the Mathematics Teacher's Specialized Knowledge Model (MTSK, Carrillo et al., 2018), the definition of mathematical competence by Blömeke et al. al. (2020) and the contributions on professional identity by Blake et al. (1998).

1.1. The MTSK model as a structure of knowledge in initial teacher training

The MTSK model has been shown to be useful as a structurer for initial teacher training programs as well as a guide in permanent training contexts (Montes, et al., 2019; Montes et al., 2021). The MTSK model continues Shulman's (1987) classical division between different teacher knowledge: mathematical knowledge and pedagogical content knowledge. The inclusion of the teacher's beliefs and the concept of specialization of the different types of knowledge that make it up are two distinctive features of this model compared to its peers (Ball et al., 2008; Rowland et al., 2009). On the one hand, it is considered that beliefs about mathematics and about the teaching and learning of mathematics permeate the different subdomains of the teacher's knowledge and serve as a filter or enhancer of professional knowledge and its development (Aguilar-González, et al., 2019). On the other hand, from the perspective

of the model, mathematical knowledge for teaching is intrinsic to the context in which it is constructed and used and, therefore, the teacher's knowledge, as a whole, is of a specialized nature (Carrillo et al., 2013). Another of the contributions of the MTSK model to the understanding of the specialized knowledge of teachers and, therefore, to the characterization of the knowledge that is built in the initial training classroom, is its detail in the description of different subdomains and categories of knowledge.

An example of how MTSK has been used to organize the content of teacher training, specifically in the subject that has been used as the context for obtaining the information in this research, can be found in Montes et al. (2019).

1.2. Skills, competencies and professional practices

During the development of a mathematics class, at any educational stage, the professional competence of the teacher plays a very important role, which has a significant impact on the quality of teaching and the learning of their students (Blömeke et al., 2020). The results, in this sense, lead our interest towards that second area of teacher training proposed by Ponte (2012), that of professional practice. We understand that initial training must be configured as a space where the construction of that knowledge in action takes place. In this way, we will consider teaching competence as the ability to offer an observable response, supported by the teacher's cognitive and motivational resources, as a result of interpreting a teaching-learning situation (Blömeke et al., 2020).

The development of this type of teaching skills allows teachers not only to respond to contingency situations in the classroom (Rowland et al., 2009), but also to be able, previously, to optimally design teaching situations and, in addition, extend their reflections throughout the process (Schön, 1991).

1.3. Being and feeling like a teacher

We understand professional identity as a part of the self-concept that develops in dialogue with the social and cultural constructions of the profession, as a result of interaction with others (Blake et al., 1998), and is considered to be in constant construction throughout of the professional career of the teacher (Losano et al., 2018). It is the way in which the individual, in our case the teacher, is personally recognized within the professional body from a social and cultural point of view. According to Lave (1988), the development of professional identity within a community and the construction of knowledge and professional practices are part of the same process. The construction of professional identity motivates, gives shape and meaning to the rest of learning. For this reason, the initial training of teachers is articulated as a key scenario in their development (Losano et al., 2018). The construction of different teaching practices and the legitimization of certain types of knowledge in the initial training classroom can help EPMs to develop a professional identity (Ponte, 2002).

2. Design

According with our interpretive point of view, we have chosen the case study as the research design (Bassey, 1999, Stake, 1995). Our intention is to understand the meanings that are developed in the initial training classroom. The research we have developed corresponds to an inductive approach, in the sense exposed by Bryman (2001). The information collection instrument that we have used in this research is non-participant observation (Flick, 2007), recorded by audio and video recording of the sessions. A total of five sessions, each lasting two hours, were recorded, in addition to PPTs class tasks.

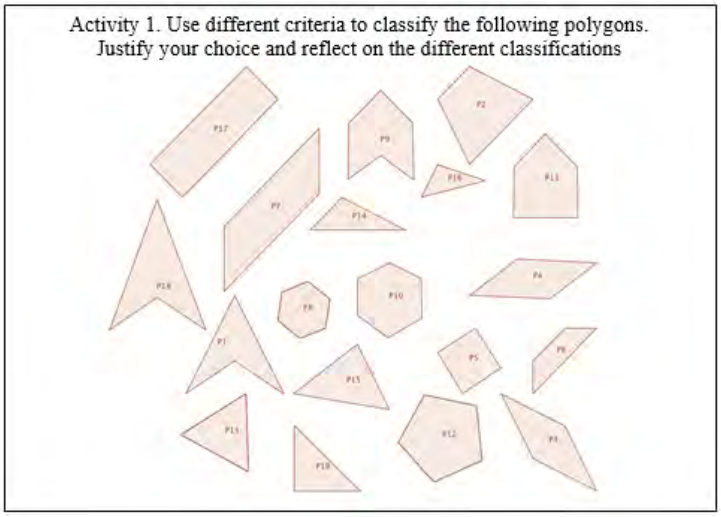
The meanings that emerge from the work in the training classroom and in the proposed training tasks have been analyzed from the point of view of content analysis (Krippendorff, 1990). The validation of the analysis is achieved through the triangulation processes of consensus among experts (Flick, 2007) and the contrast between fragments from different collection instruments, as a multi-methodological approach (Baxter & Lederman, 2001).

3. Discussion

During the analysis of the information, different types of training moments have been found in which the focus of work changes from the deepening of mathematical knowledge to the development of the professional identity of students to teacher. Below, we show some examples found.

In relation to the professional knowledge that is developed in the classroom, we find activities intended, for example, to deepen the understanding of the classification of polygons of the students for teacher PPTs (Figure 1). The objective of the activity is to deepen the understanding of the mathematical elements that underlie the classification of polygons, showing an inductive process of construction of the classification from the discussion of examples.

Figure 1. Activity on polygon classification (I).



The management of this activity allowed us to identify a particular way of working with mathematical content, which allows the PPTs to discuss and question its foundations so that they can unpack that knowledge for their teaching (Ma, 1999). In parallel, the pedagogical content knowledge of the PPTs has been developed. In this sense, the training activities invited reflection on the didactic potential of the task, its criticism and enrichment. (Figure 2).

Figure 2. Activity on polygon classification (II).

Activity 2: Analyze properties and characteristics

a) (Before class) Design a session to work on classifying triangles in 4th grade. Use some resource (geoplans, point plots, geostrips,...) for this.
b) (In class) Discuss with your classmates the strengths and weaknesses of the different proposals made.
c) Get a point plot and draw as many different triangles as you can.
d) Discuss why each one is different from the previous ones.
e) Can you draw an equilateral triangle on the plot? Reason your answer.

The observation allowed us to identify mathematical knowledge and pedagogical content knowledge that could be characterized by the MTSK model (Carrillo et al., 2018) on a recurring basis. Activities in which elements of professional knowledge were discussed as content of initial training predominated, identifying a mixed model of training in relation to mobilized knowledge (Nolla et al., 2021), over the other blocks of content determined by Ponte (2012).

The design of activities, a task that can also be seen in Figure 2, is the central teaching practice identified during the observations. However, it also reflects on the learning difficulties of mathematics in Primary Education, based on joint readings on the difficulties caused by the representation of geometric figures in textbooks (Jaime et al., 1992). and on the sequencing of geometric content based on the Van Hiele model (Fouz, 2004), which allows students to foresee contingency situations (Rowland et al, 2008) and act accordingly, including these considerations in their didactic designs.

Based on modelling strategies (Rojas et al., 2021), the teacher educator includes different didactic desings for teaching geometry within the content of the initial training, specifically in relation to the definition of the polygon, as observed in Figure 3:

Figure 3. Class transcript extract.

Teacher educator: This activity can be exported to a Primary course, as is. What we are doing here, I can transfer to Primary Education. Defining, agreeing on properties, leads us to a shared way of defining a polygon in the face of an imposed definition of which we do not understand why those criteria are imposed, right? Well, it works the same when you teach it, because your students will also be able to do this process, which is richer.

Finally, we have observed a trend towards the development of professional identity in PPTs during initial training in parallel with the rest of the training activities. Other authors account for this phenomenon (Lave, 1988). In this way, the presence of learning resource, textbook and curriculum analysis activities point towards the development of an active and critical professional profile. On the other hand, the sense of construction of mathematics and the attribution of a social character to this process, favors the development of beliefs about mathematics linked to problem solving and discovery tends. Both groups of beliefs, about mathematics and about its teaching and learning, have been studied using the MTSK model as a reference (Aguilar et al. 2019) and are part of how PPTs can be recognized within the teaching community.

4. Conclusions

In conclusion, reflecting on the initial training of mathematics teachers is a crucial issue today. The improvement of the teaching of mathematics depends on the training of mathematics teachers and this training, on how the programs are articulated in the universities. This study has served to systematically analyze the content of the initial training of mathematics teachers and contributes to initiating other interventionist-type research that can optimize the processes of initial and permanent training of mathematics teachers. The case study as a research methodology has served us to delve into the observed reality, although complementary studies are necessary to compare and discuss in different contexts.

As a consequence of the characteristics of the observed context, evidence on mathematical knowledge in initial training predominates. However, organizing initial training based on professional teaching skills is a growing approach and a reality in other training contexts. In relation to research on the processes of shaping professional identity, we find ourselves before a particularly relevant field due to the difficulty of extracting empirical evidence and the scarcity of research results in this regard.

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