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THE PRE-SERVICE TEACHERS UNDERSTANDING OF FRACTION AND HOW FUTURE INSTRUCTIONS CAN BE IMPROVED TO OPTIMISE LEARNING

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Abstract

There is an ongoing debate on whether preservice teachers should be taught the mathematics content knowledge because they start their mathematics content courses believing that they know enough mathematics to teach at a primary school level. Previous research has shown that much of the preservice teachers' knowledge lacks conceptual understanding. Consequently, the current study explored preservice teachers' knowledge of fractions. The study focuses on preservice' teachers' knowledge when comparing sizes of different fractions. The study will identify what the preservice teachers know about the comparison of size when it comes to fractions. A better understanding of how student teachers understand mathematics will inform better teaching methods for future instructions. This is to inform better instructional design in future ITE courses. The needed data consisted of 90 preservice teachers' activity scripts and a task-based interview of some students. The study was guided by the research question: What is the preservice teachers' understanding of fraction comparison, and how can future instruction be improved to optimize learning? The study adopted a mix-method approach where preservice teachers' responses to activities items were analysed from a first-year module conducted at a university level. Content analysis of the data yielded important findings that showed that preservice teachers have some misconceptions when they must determine the bigger fraction between the two. This study may be helpful to academics designing initial teacher education courses for mathematics and teachers who are already teaching mathematics in primary schools.

Keywords: *Mathematics for teaching, initial teacher education. preservice teacher practices, fractions, error analysis.*

1. Introduction

The mathematics common content knowledge is not always common to each preservice teacher. There is always an argument that the preservice teachers are assumed to possess the common content knowledge because it is based on what was previously taught during their school years. However, the pedagogical content knowledge for mathematics always lags the common content knowledge. This implies that, for an individual to teach mathematics effectively, their mathematics content knowledge should be intact. Primary school teachers' content mathematical knowledge remains an issue of broad concern in South Africa. Most of the research on the knowledge of the mathematics teacher in South Africa has been focused on in-service primary teachers, and particularly teachers at Intermediate Phase (grades 4–6) level (e.g. Carnoy & Chisholm, 2008; Taylor, 2011; Venkat & Spaull, 2015). The findings from these studies show a substantial gap in upper primary teachers' mathematical knowledge. Additionally, teacher performance on assessment items requiring reasoning beyond the purely procedural consistently shows low results (Carnoy & Chisholm, 2008).

We constantly ask ourselves whether preservice teachers should be taught the content knowledge they are going to teach to their learners, or should we assume that they already know the content? To add to that, preservice teachers also start their mathematics content courses believing that they know enough mathematics to teach at a primary school level. The study done by Stohlmann et al. (2014) confirms that prospective teachers enter universities with that belief and mindset. As a result, the initial teacher education courses in mathematics usually focus on pedagogical content knowledge (PCK) instead of the mathematics subject matter knowledge (SMK).

With a focus on primary school foundation phase mathematics preservice teachers, while zooming on the topic of fractions, comparing sizes of different fractions, the study will identify what the preservice teachers know about comparing fraction values or size to inform better instructional design in future initial teacher education courses.

2. Literature review

According to Van De Walle et al. (2012), the study of fractions is significant for students because it is essential for algebra and many advanced mathematics topics. Student teachers are taught this topic to improve their subject matter knowledge as well as the content knowledge. When students struggle to understand fractions, they will have difficulties in many areas such as decimals, percentages, rate-ratio, measuring fractions, and using them in algebra (Aliustaoğlu, Tuna & Biber, 2018). The study done by Deringöl (2019) revealed that fraction concepts are more difficult for students than many of the topics in the mathematics curriculum. The research done by Aliustaoğlu et al. (2018) showed that students at all levels of education have many misconceptions about fractions. The best way to prevent them is to focus on teacher educators because they are the ones that lay the foundations for future mathematicians.

The study done by Aliustaoğlu et al (2018) revealed that students have misconceptions about comparing fractions and operations on fractions. One of the misconceptions is based on how students view fractions. Same as the learners in the primary school level, they learn about whole numbers before they can learn fractions. Students build new concepts by using their previous knowledge of whole numbers. According to Van De Walle (2012), students use their knowledge of whole numbers on fractions and assume that numerator and denominator are distinct values without thinking that both numerator and denominator form a fraction that should be looked at holistically. For example, when finding the bigger fraction between the two, some students only focus on the numerator or denominator individually and then assume that if the number at the numerator is bigger, the fraction is also bigger, or if the number at the denominator is bigger, the fraction is smaller. This misconception is coming from the understanding that the numerator determines the size of the fraction in comparison to the whole. For example, 2/5 is bigger than 1/5, and 2/5 is bigger than 2/10. Students fail to understand the context of the fraction and overlook the concept of part and whole. The study done by Okur & Çakmak Gürel (2016) revealed that students struggle to pay attention to the fact that parts of shapes must be equal before one can even begin to compare the two fractions. This confirms what Alacaci (2014) argued when he says misconceptions are due to focusing on a single component instead of the searching relationship between numerator and denominator and thinking of fractions as natural numbers.

3. Theoretical framework

This study used students' understanding of error and misconception as a heuristic that teachers can use to inform better teaching methods for future instructions and support students to gain conceptual understanding of the mathematics they are learning (Nesher, 1987). Part of being an effective mathematics teacher is the ability to communicate mathematics common content knowledge by using effective pedagogical content knowledge that is informed by the students' knowledge. (Ball & Bass, 2000; Nesher, 1987; Shulman, 1986). This study draws from Ball & Bass (2000)'s work on mathematics knowledge for teaching and Nesher (1987)'s work on mathematics misconception by making a claim that knowing what to teach and what the learners know and how they use what they know to acquire new knowledge will determine possession of good pedagogical content knowledge.

4. Methodology

The description is based on mixed-method research. The required data were collected by analysing students' written assessments. The quantitative part focused on the number of themes obtained from activities done in class. The qualitative component is descriptive, and the researcher is the main instrument of this research describing what students know and interpret their understanding.

4.1. Selection of participants

The needed data was collected by analysing 90 preservice teachers' written activities given when the module was taught. It was a first-year module taught during the year 2021 using a hybrid learning platform. All activities were done online and student teachers' responses to the activities were analysed. Five students were also selected for a task-based interview.

4.2. Data collection and analysis

This study used two data collection methods to ensure methodological triangulation (Creswell, 2005). The first activity was a baseline assessment to discover students' knowledge of fraction size comparison. A follow-up class activity was also given to find out more about the students' knowledge, where students were requested to use the diagram to show their understanding. To triangulate the data, a task-based interview was conducted with 5 purposefully sampled students to validate their reasoning based on what they had written. According to Weber et al. (2020), the task-based interview is used to understand how mathematics students complete given tasks to gain insight into how students can be taught to complete these tasks or to discover students' thinking to support them properly to get their mathematics thinking correct.

The thematic analysis method was used to analyse data. Thematic analysis is a method used to identify, organize, and offer insight into patterns of meaning (themes) across a data set to allow the researcher to see and make sense of collective or shared meanings by identifying what is common to the way a topic is written and making sense of those commonalities (Braun & Clarke, 2012). From different collected themes, quantification through percentage was used to make sense of different identified themes. Some of the individual's chosen written work was also compared to what the same individual explained during interview and the conclusion was drawn from that.

5. Findings and discussions

5.1. Activities

From the first activity, students were asked the following question: Tom claims that ¾ < 5/8 because 3 < 5 and 4 < 8. Is Tom correct? How can you help his understanding with the use of representation/s? From the answer given, the following data was received in the table below:

Table 1.

Themes	**Number of students**	**Percentage**
Students who knew that Tom was wrong	72/90	79%
Students who think Tom is correct	18/90	21%
Students who could not explain or support their reasoning	55/72	76%
Students who supported their reasoning using diagram	12/72	17&
Students who supported their reasoning by converting fraction to decimal	6/72	8%
Students who supported their reasoning through solving equivalent fraction	17/72	19%

From the table above, it is clear that students have some knowledge about fraction size. 79 % of the students knew which fraction was bigger between the two. However, out of the 79%, 76 % could not give a correct explanation why the other fraction was bigger than the other. These are students who explained incorrectly or students who did not give a reason. To validate what was obtained in the first activity, a similar question was given with direct instruction to see if students could support their reasoning using different interpretations. Students were given the activity in the form of a three-day assignment. The question from the activity was as follows: Vuyo claims that 3/ 4 < 6 /9 because 3 < 6 and 4 < 9. Is Vuyo correct? Use three different methods (algorithm, diagram, and a practical example) to make your claim plausible.

Table 2.

Themes	Number of students	Percentage
Got the diagram correct	38/90	42%
Got the diagram wrong	52/90	58%
Got diagram wrong because the whole was not the same size	26/52	50%
Got the diagram wrong because parts of the whole were not the same size	12/52	23%
They were completely wrong	12/52	23%
Students who said Vuyo is correct	2/52	4%
Students who got the algorithm correct	74/90	82%
Students who got the practical example correct	11/90	12%
Students who got the practical example wrong	79/90	88%

More students seem to understand the difference between fraction size from the table above. However, students did not know all the concepts based on fractions size. It appears that students master the concept of a written algorithm where the denominators are made to be the same size first and then compare the size of their denominators. The fact that 58% of the students got the diagram wrong shows that most of the students do not fully understand the concept of fractions. Out of those who got the diagram wrong, students don't understand the idea of a whole. As much as they understood the algorithm part that stipulates that to compare fractions, you must first make the denominators the same and then compare the numerators, they were unable to transfer this knowledge to the use of a diagram. Making denominators the same means having equal parts inside a whole. 23% of the students did not understand this concept. Having an equal whole means we are comparing the same thing. When the fraction being compared have a different whole, it is like comparing bread and apple, which is not easy to compare because bread and apples aren't usually the same size. 50% of the students also did not understand this concept because they were drawing two diagrams that were not the same size.

5.2. The interview

From the interview, student A did not agree with Tom. This was her statement: "Tom is incorrect, 3/4 is > 5/8 because we are dealing with fractions. In fractions, the denominator represents how many parts there are in the fractions and in this case 4 will be greater than the 8" The question that was asked during the interview was to elaborate on her reasoning. The student replied by saying: "In fractions, small number fractions are greater than big number fractions, and when it comes to whole numbers, the small numbers remain small, and big numbers remain big" This shows that the student does not fully understand fractions. Student A has some truth that is not complete. The student is looking at a fraction as number and not as a representation of a part of a whole. Once we start manipulating the numbers (elements of fractions) without referring them to their representation, we create learners who do not understand the true meaning of fractions. Hence most of the students manipulated the fractions by making the denominator the same. This is all just procedural without focusing on conceptual understanding.

Another student called Student B wrote: Tom is incorrect, 3/4 is > 5/8 because we are dealing with fractions. In fractions, the denominator represents how many parts there are in the fractions, and in this case, 4 will be greater than 8. When asked to elaborate during the interview, she said, "The bigger the denominator, the smaller the quantity, because there have been more pieces cut in the higher denominator."

This shows misconceptions. What the student is saying is not always true. When comparing fractions of the same size whole, the bigger denominator will reduce the size of the fraction. This principle does not apply when the whole is not the same size. Student usually misconceive this concept and use it out of context.

6. Conclusion

This study reveals that students know how to compare fractions using a written algorithm but lack conceptual understanding of the representation of the written algorithm on the diagram or representing it using practical examples. Preservice teachers also have the same misconception as the learners in primary school. This study recommends that initial teacher educator institutions should not assume that students know but teach all the concepts from the beginning to cover all the loopholes that might be there.

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