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STUDENTS' BEHAVIOR ON A "MATHEMATICAL LITERACY" PROBLEM

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Abstract

The construct of mathematical literacy, which has become more popular through OECD/PISA programme (Programme for International Student Assessment) recognizes the importance of students' capacity to reason mathematically, use and understand mathematical concepts while they explore real world problems. According to PISA's results, the performance of Greek 15-year-old students in mathematical literacy has not improved and has remained below the OECD average throughout the years it participated in PISA, since 2000 till today. Except PISA's general and comparable results on students' performance, students' responses on open constructed-response items and most of the items themselves are not available on PISA's open database. As a consequence, researchers cannot study them in order to analyse the way that students in Greece manage such context-based mathematics tasks and cannot comment on their difficulties which could be used as evidence to justify their steadily low performance all these years. This current study aims to explore and examine in depth the way students, who are completing compulsory education in Greece, respond in a real mathematical problem encompassing a lot of characteristics of PISA's math problems. Therefore the open constructed-response problem, which belongs to the "uncertainty and data" content category as this described in PISA's mathematics framework, was given to 650 students who complete the compulsory education in Greece (9th Grade). After decoding their answers, we comment on the different ways of dealing with the real context-based mathematics problem and clarify students' difficulties.

Keywords: *PISA, mathematical literacy, Greece, data and uncertainty.*

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1. Introduction

"Mathematical literacy is an attribute that is on a continuum and with the potential for growth always present" (OECD, 2018b). According to mathematics' framework for the needs of the Programme for International Student Assessment, PISA 2022, mathematical literacy is defined as "an individual's capacity to reason mathematically and to formulate, employ and interpret mathematics to solve problems in a variety of real world contexts" (OECD, 2018b). This notion of mathematical literacy supports both the importance of developing a deep understanding of pure mathematical concepts and the benefits of engaging in exploration of the abstract world of mathematics (OECD, 2004; OECD, 2013; OECD, 2019). As long as there are plenty of problems and situations in real life in which the understanding of mathematics and mathematical reasoning is required, it is very important for students to have the ability to treat mathematics as a critical tool (OECD, 2013). For that reason, the extend and the way that students react on such situations and problems are important to be explored, with the ultimate goal of finding ways to develop more characteristics of mathematically literate students and individuals in general.

From 2000 till today, Greece's mean performance in mathematical literacy on Programme for International Student Assessment (PISA) has been consistently below the OECD average, with an average difference from it, of around 40 score points which corresponds to one whole school year (Nolka & Sofianopoulou, 2021; Nolka & Sofianopoulou, 2022). This stable and low status may be partly justified by the poor alignment of Greek mathematics curricula and junior high school mathematics textbooks with the PISA mathematics framework and their strong content orientation (Nolka & Sofianopoulou, 2021; Nolka & Sofianopoulou, 2022; OECD, 2018a; IEP, 2019). But apart from this theoretical explanation derived from curriculum, a different way to explain why students in Greece do not respond very successfully to real problems or why they don't concentrate so many characteristics of a mathematically literate person, is to examine the ways in which they deal with such problems and to explore the difficulties they face and what factors might be hindering this development of mathematical literacy.

One of the interrelated aspects that mathematical literacy is analysed in PISA is the mathematical content. The four content categories are: change and relationships, space and shape, quantity and uncertainty and data. In this research we choose to analyse a problem that belongs to the "uncertainty and data" category. According to statistics educator David S. Moore, the uncertainty *"recognizes the importance for students to view data as numbers in a context"* (OECD, 2009). According to the mathematics curriculum for lower secondary education in Greece, which includes the single cross thematic curriculum framework (DEPPS) and the detailed curricula (APS), the percentage of teaching material that deals with topics and concepts that could be included in the broader category of "uncertainty and data" is approximately 8% (P.I., 2011) in proportion to the rest of the curriculum. This element makes the results of the present research even more interesting, given that the students in the sample are not familiar with problems of such a mathematical content category.

2. Method

The participants of this study were 650 students from 19 high schools who are completing the compulsory education in Greece. The objective of the research was to explore and examine in depth the way students respond to this realistic context–based problem and at the same time to investigate students' difficulties.

Table 1. The "uncertainty and data" problem.

Last year, the planet was hit by a flu pandemic. By the end of last December the number of the outbreaks and deaths from the pandemic in five countries, appear in the table below.

Country	Population	Outbreaks	Deaths
A	10,700,000	2,530,000	36,624
B	10,720,000	1,540,000	21,479
C	11,560,000	2,290,000	28,518
D	67,220,000	14,600,000	150,000
E	83,240,000	7,550,000	114,000

Looking at the table above, Vasia claims that country E had a higher rate of deaths than country A. Is Vasia right? Yes or No? Justify your answer.

The given real world problem was a subject allocated to the "uncertainty and data" content category according to PISA's programme mathematical framework, but at the same time it was aligned to the Greek high school mathematics curriculum. The item required calculation and interpretation of data on death rates. Moreover, it required the knowledge of the basic row-column conventions of a table, as well as data – handling ability in order to choose and manage the appropriate data. Students from one side could follow all the mathematical processes of the modeling cycle described in the definition of mathematical literacy (formulate, employ and interpret) in order to relate the real context of the problem to their familiar mathematics and come up with an acceptable solution and answer. On the other hand, it wasn't necessary for students to engage all the stages of modelling cycle but the dominant process for the specific real-problem of this research which needed to be applied was the process of interpretation, which also encompasses the notions of application and evaluation. During this process students had to interpret, apply and evaluate their mathematical outcome and at the same time it's reasonableness in the context of the realistic problem itself.

3. Results and discussion

Table 2. Students' performance on the problem.

	Frequency	Percent	Cumulative percent
Correct answer	220	33,8%	33.8%
Partial correct answer	103	15.8%	49.6%
Wrong answer	266	41%	90.6%
Missing	61	9.4%	100%
Total	650	100%	

Almost half of the students responded with a correct or a partially correct way to this real - world problem. Moreover, 1/3 of the students responded successfully to the problem, by following all the needed mathematical processes in the correct way while the 2/5 of the students failed to find the right answer in an acceptable and justifiable way.

Table 3. Frequency and percentages on students' coded answers.

Answer	Description	Frequency	Percent
Correct answer – code 2.1	Selects "No" and provides acceptable numerical values or numerical analysis for both countries	107	16.5
Correct answer – code 2.2	Selects "No" and provides an acceptable and appropriate analysis of the data without using numerical values.	113	17.4
Partial Correct answer – code 1.1	Selects "Yes" with supporting a "No" answer with partially correct explanation.	49	7.5
Partial Correct answer – code 1.2	Selects "No" and provides an explanation with or without numerical values, on one of the two countries.	20	3.1
Partial Correct answer – code 1.3	Selects "No" or "Yes" and compare the deaths with the outbreaks instead of the population.	34	5.2
Wrong answer – code 0.1	Selects "Yes" and compare only the number of the deaths.	133	20.5
Wrong answer – code 0.2	Selects "Yes" or "No" with an incorrect explanation.	39	6
Wrong answer – code 0.3	Selects "Yes" or "No" without explanation	94	14.5
No answer	Missing	61	9.4
Total		650	100

In a more detailed analysis of the students' responses, we coded them, grouping the responses for further comment on the way they approached the questions. The correct answers were grouped into two categories which were given by approximately the same percentage of students in the sample. In the first correct answer category/code, students had successfully incorporated all steps of the mathematical modeling cycle into their written answer. More specifically, they first recognized the mathematical nature of the problem and succeeded in formulating it in mathematical terms. Subsequently, they solved the problem using mathematical concepts and procedures successfully applying the process of employ. During the last step, students evaluated their solutions and interpreted them in a right way within the original and given real-world situation. Compared to the first category of acceptable answers in the second category/code of correct answers, not all procedures of the modelling cycle were used clearly. On the other hand, the final processes of interpretation and evaluation, which were allocated the problem, were very clearly written and correct, creating a fully justifiable answer in the context of the real - world problem. In both correct categories students showed an ability to handle percentages and fractions and their engagement with the processes of interpretation and reasoning.

Partially correct responses were grouped into three distinct subgroups. In the first one of them, category/code 1.1, students recognised the mathematical nature of the situation and formulated it in mathematical terms (correct or semi-correct), applying mathematical concepts (correct or with some arithmetical or mathematical mistakes) to solve the mathematically-formulated problem while in the end failing to evaluate correct or not at all the reasonableness of their mathematical solution in the context of the real-world problem. In the second category of partial correct answers an interpretation of the solution was used but not for both comparable countries and also without clear use of the previous procedures of formulation and employ. The category/code 1.3 of partial correct answers included the answers in which students followed the whole procedure of formulating, employing and interpreting but the students compared the values from the table-column deaths with the values of the table-column outbreaks instead of the population.

Furthermore, three subcategories were distinguished among students' incorrect answers. The majority of the students who gave wrong answers didn’t identify the significant variables or recognise a mathematical structure, but they only compared the numbers themselves on the one table-column (deaths), without any formulation or comparison with the values from the other table-column (population). Students in this category cannot interpret or either recognise the situation in context but they only used a direct and wrong inference. On the second category/code 0.2, students used the formulation and/or the application with wrong way and they were not capable of making correct or logical interpretation of their results while at the same time they made a lot of mathematical or arithmetical mistakes. On the third incorrect answer category/code 0.3, students did not explain their answers or they did not record any of their thinking, revealing their difficulty to justify their answers in words.

Among all the preceding subcategories of correct, partially correct and incorrect answers, the largest percentages were concentrated in the first incorrect answer category (0.1) and they were followed by the two correct answers categories (2.1 and 2.2) with very similar percentages.

4. Conclusion

Mathematical literacy is assessed in the context of a challenge or a problem that arises in the real world (OECD, 2014). Students in Greece, including the students in the sample of this research, according to Greek mathematics curriculum, are not systematically trained in the mathematical modelling cycle and more specifically on the mathematical processes formulate, employ and interpret, which are needed to connect the real context of the problem with the mathematics in order to solve the problem. Furthermore, students were largely unfamiliar with mathematical concepts that contain features of the category “uncertainty and data”. Therefore, the ways they reacted, solved and answered the problem in this study were based basically on their general education in mathematics throughout the duration of their compulsory education.

The results of this study, which can be characterized as a positive percentage, showed that almost the half of the students were able to use some or all the mathematical processes of the mathematical modelling cycle in a correct or a semi correct way in order to solve this uncertainty and data context based and real problem. Most errors, apart from numerical errors, were traced to the students' inability to extract the essential mathematics to analyse and solve the problem, that was to formulate and moreover to their inability to reflect upon a mathematical solution, which is linked to the process of interpretation.

Exposing students in the mathematics classes to more real and context-based problems, and recognizing and familiarizing themselves with the process of mathematical modeling are some of the requirements that could help students improve their mathematical literacy skills. It may therefore be helpful for teachers to activate students and also motivate them in this direction but also for teachers themselves to be trained through specific training programs in the same direction. In addition, all these suggestions could be implemented more easily if they were included in the curriculum and in students’ mathematics textbooks. A positive element is that a new curriculum (IEP, 2023) in mathematics for all compulsory education in Greece, which is already in a pilot application, is going to be in effect within the next school years, giving greater emphasis on the fields of statistics and probability and as a consequence, more problems of uncertainty and data content category will be contained. So, it will be challenging to study the results that will bring this new curriculum in the direction of improving students’ mathematical literacy abilities in the future.

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