https://doi.org/10.36315/2023v2end105
Education and New Developments 2023

ORDINARY DIFFERENTIAL EQUATIONS IN A MATHEMATICAL MODELING CONTEXT

Amábile Mesquita[1], & Celina Abar[2]
[1]Pontifical Catholic University of São Paulo (Brazil)
[2]Department of Mathematics, Pontifical Catholic University of São Paulo (Brazil)

Abstract

Many of the problems involving motion, growth rate, electricity, physical, biological phenomena, or that in the context of rate of variation are modeled with Ordinary Differential Equations (ODE). This content provides several relationships between Mathematics and other Sciences, offering opportunities for contextualization and creation of an environment conducive to learning. Researches indicates that the approach given in the teaching of ODE has a strictly algebraic approach, where students learn the methods of resolution without focusing on the behavior of the solution obtained, without performing a qualitative analysis of the problem that generated these equations. This paper aimed to present and analyze a teaching and learning experience that took place during the first classes of Applied Differential Equations in a Mathematics Teaching Degree course using the GeoGebra software as a pedagogical resource. The classes took place in the computer lab of the State University of Goiás, in the city of Goiás, with the participation of 10 students, and included activities designed from the perspective of Mathematical Modeling as a teaching strategy. The theoretical framework was based on reflections on Raymond Duval's Theory of Registers of Semiotic Representation, which considers that learning occurs when the individual performs articulations between the different representation registers of the same mathematical object. As a result, we found that mathematical modeling and GeoGebra provided a favorable environment for teaching and learning, as they allow the integration of theory and practice. The sequence of activities, prepared in a context in which the students experience, contributed to the students' involvement in their development.

Keywords: *Ordinary Differential Equations, Mathematical Modeling, degree course in Mathematics, GeoGebra, teaching and learning.*

1. Introduction

Many of the problems involving motion, growth rate, electricity, physical and biological phenomena, or those involving any variation rate are modeled with Ordinary Differential Equations. Therefore, this content provides various relationships between Mathematics and other Sciences, offering opportunities for contextualization and creation of an environment conducive to learning.

This research on the use of mathematical modeling as a teaching and learning strategy for Differential Equations (DE) in a Mathematics Teaching degree program, was motivated by research such as Dullius, Veit and Araujo (2013), Alvarenga, Dorr and Vieira (2016) and Rosa, Alvarenga and Santos (2018), which indicate that the approach adopted in teaching this subject focuses strictly on algebra and allows students to learn resolution methods without focusing on the behavior for the solution obtained, i.e., without performing a qualitative analysis of the problem that generates these equations, and with difficulties in perceiving the existing connection between the DE and the real modeled system and interpreting its terms.

Therefore, this research aimed to investigate the potential of a sequence of activities, developed from a mathematical modeling perspective, with the support of the GeoGebra software and based on the Theory of Registers of Semiotic Representation of Duval (1993), in the DE teaching and learning process, more specifically, in the qualitative analysis of DE solutions.

Using a mathematical modeling is necessary, as it allows for interaction between the "real world" and mathematics, as stressed by Bassanezi:

> Mathematical Modeling is a dynamic process used to obtain and validate mathematical models. It is a form of abstraction and generalization for the purpose of forecasting trends. Modeling essentially consists of the art of transforming real situations into mathematical problems whose solutions must be interpreted in usual language. (Bassanezi, 2022, p. 24, translation by the authors).

Dullius's et al. (2013) research provides evidence that mathematical modeling can facilitate learning, as teaching activities in a modeling environment bring about various mathematical and non-mathematical concepts, which favors learning.

Mathematical modeling, according to Bassanezi (2022), must follow a sequence of steps: Experimentation: step where data is obtained, whether qualitative or numerical; Abstraction: procedure that leads to the formulation of mathematical models, wherein the selection of variables, problematization, formulation of hypotheses and simplification are established; Resolution: linked to the degree of complexity used in the formulation, may be analytical or numerical, and can only be made possible through computational methods; Validation: the process of accepting or rejecting the model; Modification: if the model is not accepted, the problem data must be reviewed to then the necessary adjustments must be made in order to redo the steps until a better approximation of reality is obtained.

To support this teaching methodology, the use of Information and Communications Technologies (ICT) was considered, as it is in the development of classes that "[...] one realizes that ICT allows that activities be performed that would have been impossible with the use of paper and pencil only; it allows the organization of pedagogical situations with greater potential for learning" (Marin, 2008, p. 138, translation by the authors).

For this research, the GeoGebra Software was used, as it has many potentialities, such as its dynamism, its development aimed at teaching and learning Mathematics in its various teaching levels (from elementary school to university), it is free and easily accessible on the internet, and available in several languages, thus favoring its use. Also, GeoGebra has the didactic benefit of simultaneously presenting several representations of the same object.

The theoretical framework that guided the development of the teaching practice of this paper was Raymond Duval's Theory of Registers of Semiotic Representation. This theory clarifies the importance of using semiotic registers for mathematical learning due to the characteristics of mathematical objects that are abstract and need to be represented.

Semiotic representations

> are the product of using signs that belong to a representation system and have their own restrictions of meaning and functioning. (Duval, 1993, p. 39, translation by the authors).

According to Duval (1993), learning occurs only when the student manages to articulate naturally between the various registers of representations referring to the mathematical object.

There are three cognitive activities that characterize semiotic representation registers. The first one is the formation of an identifiable representation that can be determined through a comprehensible sentence, a drawing, a figure, a formula, among others. The second one is the treatment of a representation, which is the transformation thereof into the register itself, e.g., algebraically solving the DE. When the representation of a mathematical object is transformed into another representation, the conversion takes place, such as a graphical representation obtained from a given function. Therefore, the treatment takes place within the same register and the conversion takes place between different registers.

This research was developed using a qualitative paradigm, highlighting the present interpretative/subjective process, understanding that people act based on their beliefs, perceptions, feelings, and values, with behaviors full of meaning and senses that are not immediately assimilated, but unveiled (Alves-Mazzotti, & Gewandsznajder, 1998). Santos Filho and Gamboa (2009) mention that the focus of qualitative research is the individual experience of situations, the process of construction of meanings, common sense, the "how". Qualitative researchers originate from phenomena that interest them.

In addition, seeking to achieve the desired objectives, field research was carried out, which is considered a "[...] type of investigation in which data is collected directly in the place where the problem or phenomenon occurs" (Fiorentini and Lorenzato, 2012, p. 106, translation by the authors). In this case, it was the university, where "the researcher plays the role of observer and explorer, directly collecting data in the place (field) where the phenomena have occurred or appeared," as pointed out by Barros and Lehfeld (2005, p.75, translation by the authors).

2. Research development

This research was conducted with the participation of 10 students of the Mathematics Teaching Degree at the State University of Goiás, in the city of Goiás, Brazil. Classes took place in the computer lab, made available by the institution. The computers used by the students had the GeoGebra software previously installed. The researcher played the role of teacher/researcher for 12 days with the authorization of the head teacher of the class and the course coordinator. The meetings lasted 3 hours each, and took place on Tuesday evenings in September, October, November and December 2022. It should be highlighted that the students were in the 6th period of the course, and that the first four periods were carried out remotely due to the COVID-19 pandemic.

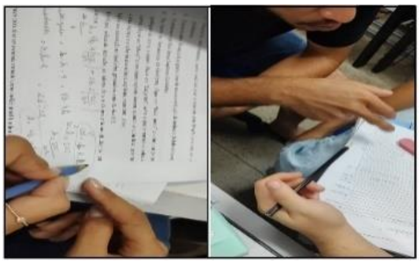
The students had already been informed that during the first classes there would be a teacher/researcher teaching the discipline of Applied Differential Equations, and that the course syllabus would not be negatively affected.

At the first meeting, the course plan was presented and the respective topics on DE were informed, i.e., definition, importance, where to use, how they emerged, terminology, types, examples, classification, field of direction, solutions, qualitative analysis of solutions, some methods of solving and initial value problems. The students were invited to answer the questionnaire “Getting to know the research participant” containing questions about mathematics in high school, the motivation for choosing the course, derivation, and integration, GeoGebra and DE, in addition to the authorization for participation in the research. As most students did not know GeoGebra, they were asked to access the software and perform some activities for their familiarization. As pedagogical support, a projector was used with which the activities were presented. Figure 1 illustrates the students during their first class.

Figure 1. Solving Preliminary Activities in GeoGebra. Source: Research Data.

After reviewing the questionnaire, we found a gap in the students’ knowledge of derivation and integration, knowledge which is of paramount importance for the teaching of DE. Therefore, in the following classes, we conducted activities on the necessary content, considering a proposal by Almeida (2017), who presents a material for teaching calculus containing applets produced in GeoGebra. For the analysis of the activities, students were asked to work in groups, favoring interaction, the development of strategies for resolution and the discussion of solutions. These activities consisted of situations present in the daily life of the participating students, i.e., civil construction activities, commerce, and rural production activities. These activities were developed using mathematical modeling steps, where data were provided by the problems (Experimentation), models were formulated (Abstraction), Resolved (Resolution), solutions were Discussed, i.e., the resolution was socialized (Validation). Some students realized that their models were not in line with reality and, therefore, made the necessary adjustments, including or excluding parameters (Modification), as shown in Figure 2.

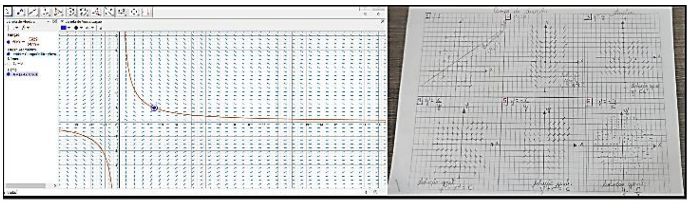
Figure 2. Discussing Derivation and Integration activities. Source: Research Data.



In one of the activities, they were requested to build a cylindrical tank that could capture 1000 liters of rainwater with dimensions that minimized the cost. During the solution socialization, which everyone used GeoGebra to better understand the result, one of the students, who works in civil construction, said that such a solution was impossible. From this observation, they started a discussion to find which the formulation problem was, and then they perfected the model. In this activity, the students used the three cognitive activities that characterize registers of semiotic representation: the formation of an identifiable representation, they formulated the problem; the treatment, they solved it algebraically; and the conversion, they used the software to build the graphical representation of the problem and then confirm whether the geometrically obtained solution was appropriate.

After the derivation and integration activities were performed, the study on DE began. In the first classes, the field of directions was built on squared paper, and then using SlopeField command in GeoGebra to compare the resolutions and check the facilities provided by the software, as shown in Figure 3.

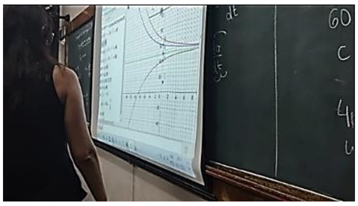
Figure 3. Building the field of directions. Source: Research Data.



p-ISSN: 2184-044X e-ISSN: 2184-1489 ISBN: 978-989-35106-4-3 © 2023

The first DE contextualized activities were classic problems, used in textbooks and which already provide all the data in the statement. The modeling steps were fulfilled as the students recognized the data, formulated the problem, solved it using the field of directions that was built in GeoGebra, and validated the model. In all DE resolution activities, using some methods, students compared the analytical solution with the geometric solution obtained from the software. Before solving it analytically, they already predicted the results through the geometric resolution offered by GeoGebra, as seen in Figure 4.

Figure 4. Solving DE geometrically and analytically. Source: Research Data.

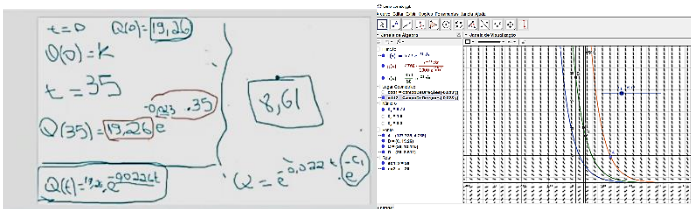


As a final activity, students were asked to choose a topic so that a problem and its resolution could be proposed from it. After several meetings and research, they chose to work with the topic of the radioactive decay of Cesium-137, a radioactive accident that occurred in Goiânia in 1987, a city close to the city where they study, and which was reported internationally. For this, the problem data were obtained from the internet. Students researched the half-life of Cesium-137, which is approximately 30.2 years, and the starting amount of Cesium-137, which is approximately 19.2 grams, to model the problem. Their goal was to discover the current amount of Cesium-137 that still exists in the city of Goiânia. These discussions on the subject are in line with what Bassanezi (2022) proposes about modeling:

> Efficient mathematical modeling allows one to make predictions, make decisions, explain and understand; finally, participate in the real world with the ability to influence its changes. We stress once again that the applicability of a model depends substantially on the context in which it is developed – a model can be "good" for a biologist, but not for the mathematician, and vice versa. (Bassanezi, 2022, p. 31, translation by the authors).

Each group formulated the problem as they pleased, and then shared their proposals. They solved it analytically and graphically and then validated it, whereupon they compared their results and decided on the model that best represented the problem, as shown in Figure 5.

Figure 5. Deduction of the formula and resolution of the DE that represents the Cesium-137 problem. Source: Research Data.



Considering the table with the data representing the amount of cesium-137 over the years, the FitImplicit and FitExp commands in GeoGebra were also used to compare the graphs with those previously obtained through commands SlopeField and SolveODE to observe the similarity between both. Figure 6 presents this new construction and the contribution for the comparison of results, representing the conversion of the semiotic registry according to Duval (1993).

Figure 6. Using the FitImplicit and FitExp commands in GeoGebra. Source: Research Data.

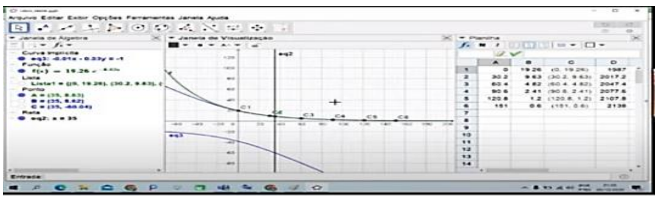


Table 1 presents the formulations and results of the algebraic and geometric resolutions obtained through the various commands used. The *x* representing how many years have passed since the radioactive accident and *y* representing the current amount of Cesium-137 that still exists in the city of Goiânia, representing the treatment of the semiotic registry according to Duval (1993).

Table 1. Formulations and results. Source: Research Data.

FitImplicit	$-0.01x - 0.03y = -1$	$x = 35\ y = 8.62$
FitExp	$f(x) = 19.26e^{-0.002x}$	$x = 35\ y = 8.63$
SolveODE = Algebraic formulation	$h(x) = \frac{963}{50} e^{-23\frac{x}{1000}}$	$x = 35\ y = 8.611$

3. Final remarks

This paper aimed to present and analyze a teaching and learning experience that took place during the first classes of Applied Differential Equations from the perspective of mathematical modeling and based on the Theory of Registers of Semiotic Representation.

Through the theoretical constructs considered, an environment conducive to teaching and learning, to be developed during DE classes, was provided. It is worth mentioning the difficulties presented by the students in relation to the basic contents of mathematics and, also the difficulties of some of them faced with the use of computers. GeoGebra could be used on cell phones, but few did so on their respective devices.

GeoGebra provided agility, practicality, and another resource to represent the problem, in the case of cesium-137, making it possible to compare the results.

The proposed activities provided a favorable environment for the use of mathematical modeling which students followed its steps. When reviewing content or when developing activities in the context of DE, the three cognitive activities took place according to Duval's theory (1993) used: formation of an identifiable representation, treatment, and conversion.

References

Almeida, M. V. *Material para o ensino do cálculo diferencial e integral: referências de Tall, Gueudet e Trouche*. (2017). 261 f. Tese (PHD in Mathematics Education) - Programa de Estudos Pós-Graduados em Educação Matemática, Pontifícia Universidade Católica de São Paulo, São Paulo, 2017. https://repositorio.pucsp.br/jspui/handle/handle/20263.

Alvarenga, K. B.; Dorr, R. C.; Vieira, V. (2016). O ensino e a aprendizagem de cálculo diferencial e integral: características e interseções no Centro-Oeste brasileiro. *Revista Brasileira de Ensino Superior*, v. 4, n. 2, pp. 46-57.

Alves-Mazzotti, A. J.; Gewandsznajder, F. (1998). *O Método nas Ciências Naturais e Sociais*: Pesquisa Quantitativa e Qualitativa. São Paulo: Pioneira.

Barros, A. J. P.; Lehfeld, N. A. S. (2005). *Projeto de Pesquisa: propostas metodológicas*. 16. ed. Petrópolis, RJ: Vozes.

Bassanezi, R. C. (2022). *Ensino-aprendizagem com modelagem matemática*. 4. ed. São Paulo: Contexto.

Dullius, M. M.; Veit, E. A.; Araújo, I. S. (2013). Dificuldade dos Alunos na Aprendizagem de Equações Diferenciais Ordinárias. *ALEXANDRIA Revista de Educação em Ciência e Tecnologia*, v.26, n.2, 207-228.

Duval, R. (1993). Registres de représentation sémiotique et fonctionnement cognitif de la pensée. Annales de Didactique et Sciences Cognitives, *IREM-ULP*, v. 5, p.37-65, Strasbourg.

Fiorentini, D.; Lorenzato, S. (2012). *Investigação em educação matemática:* percursos teóricos e metodológicos. 3. Ed. rev. Campinas, SP: Autores Associados.

Marin, D. (2008) *Professores de matemática que usam a tecnologia de informação e comunicação no ensino superior*. 95 f. Dissertação (Master in Mathematics Education) – Universidade Estadual de São Paulo "Júlio de Mesquita Filho", Rio Claro.

Rosa, C. de M.; Alvarenga, K. B.; Santos, F. F. T. (2018). *Desempenho Acadêmico em Cálculo Diferencial e Integral*: um Estudo de Caso. Universidade Federal de Goiás.

Santos Filho, J. C.; Gamboa, S. S. (2009). *Pesquisa Educacional: quantidade-qualidade*. 7° edição, São Paulo: Cortez.