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PRE-SERVICE TEACHERS' KNOWLEDGE BASE FOR TEACHING MATHEMATICS: THE DEVELOPMENT OF SYNTACTIC AND SUBSTANTIVE KNOWLEDGE

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Abstract

The purpose of this study is to analyze pre-service teachers' knowledge base in terms of their substantive subject matter knowledge and syntactic subject matter knowledge. The study involves 20 students from a program course for K-3 teachers, *Advanced Math Development* (Semester 6) and 25 from a program course for teachers in Grades 4-6, *Mathematics 4* (Semester 6). The data collection includes pre-service teachers' reflections upon their written analysis of teaching observations as mathematical content and written examples of how to implement a teaching moment (content) that they had created themselves. The crucial content for the study is algebraic patterns, rational numbers (fractions), proportionality, and combinatorics. The findings from the study identify and highlight areas to be developed, especially with respect to student teachers' knowledge base and what they need to explore concerning subject-specific content for teaching mathematics, as well as crucial relevant content (substantive knowledge) and how to apply this in teaching (syntactic knowledge). Generally speaking, the study highlights the challenges student teachers face in grappling with mathematical concepts when they describe their observations, as well as in their own construction of a teaching moment. This means that in teacher education for mathematics, more attention ought to be paid to developing a knowledge base founded on the transformation of substantive knowledge to syntactical knowledge and its impact on student teachers' learning of mathematics for teaching purposes. The study can also provide a deeper understanding of student teachers' learning process and challenges related to the knowledge base, e.g., its substantive and syntactic components.

Keywords: *Pre-service teachers, knowledge base, substantive subject matter knowledge, syntactic subject matter knowledge, mathematics for teaching.*

1. Introduction

Pre-service teachers' knowledge of mathematics is an important issue in mathematics education and has been an active research field over the last decades. Teachers' mathematical knowledge, as well as its conceptual meaning with regard to what student teachers should learn, has been described by such researchers as Shulman (1986) and Ma (1999). They developed a basis for research orientation and constructed the theoretic model of Pedagogical Content Knowledge and its extension, via Subject Matter Knowledge, to Mathematical Content Knowledge for Teaching. Empirical studies of teachers' knowledge claim that pre-service teachers must transition from a student's perspective to a teachers' perspective in order to understand mathematical content themselves and grasp how students learn the same content based on different abilities, knowledge, and conditions for development. Pre-service teachers also learn mathematics and mathematical content in different ways, such as education activities, teaching evaluation, and the construction of teaching activities. According to Dewey (1998), each learner's knowledge base is founded on reflection and the analysis of situations, which they carry out themselves on the basis of their own experiences. The same starting point is described by other researchers with regard to teachers' subject-related, content-specific professional development via critical reflection and self-reflection (Mezirow & Taylor, 2009; Hardy, Decristan, & Klieme, 2019). Against this background, pre-service teachers' reflections related to practice and analysis of the teaching experiences of their peers is a particularly important part of their own development of a mathematical identity. According to researchers, a continuous connection between the development of student teachers' knowledge of mathematics and teaching practices ensures a relationship with the subject of mathematics that can positively impact their learning of mathematics (Boaler, 2002). The construction of knowledge from practical examples of

teaching supports pre-service teachers in identifying the relationship between practice and theoretical knowledge. At the same time, the evaluation of practice presupposes a theoretical knowledge that Shulman (1987) called "scientific knowledge for teaching as a diverse disciplinary knowledge and its interrelations as a framework for a knowledge base for teaching." In the context of pre-service teachers' education, the study of knowledge bases for teaching is important and timely, both for research regarding pre-service teachers' learning for the purpose of teaching mathematics and for current research discussions about teaching and its impact on student learning (Darling-Hammond & Bransford, 2005; Hordern & Tatto, 2018). Leikin and Zazkis (2010) posit that there is both a subject content knowledge component and an epistemological knowledge component to students' learning. A successful combination of these components can be achieved by a learning discourse for pre-service teachers in which their subject matter knowledge within specific mathematical fields can serve as a logical starting point for constructing teaching activities and conceptual structures for teaching. According to researchers, a responsive relationship between subject mathematics knowledge field and teaching provides an understanding of what to teach and how to teach it, as well as principles for students' learning and understanding of how this contributes to students' continuous and progressive learning. This kind of knowledge is an increasingly important part of pre-service teachers' preparation for their future profession as teachers, both in terms of learning what constitutes subject matter and with regard to mathematical concepts and the connection between mathematical concepts related to teaching and learning mathematics (Jakobsen, Thames, & Ribeiro, 2013; Norton, 2019). This study examines pre-service teachers' learning of subject matter and its conceptual meaning with regard to Shulman's theory of substantive and syntactic subject matter knowledge structure.

The purpose of the study is to analyze pre-service teachers' knowledge base and its development related to substantive subject matter knowledge and syntactic subject matter knowledge.

2. Theoretical background and starting points

The theoretical framework refers to the conceptualization of teachers' content knowledge for teaching, as defined by Shulman (1986, 1987) and consistent with the constitution of pedagogical content knowledge and subject matter knowledge (Ball, Thames, & Phelps, 2008). According to these researchers, as a practical realization of knowledge, skills and competencies for teaching, pre-service teachers' content knowledge leads to professional prosperity and expertise via theoretical study and practical experiences. To underline the relationship between them, pedagogical content knowledge, subject matter knowledge, and subject-specific content knowledge were defined as categories for a knowledge base (Shulman 1986), both in theoretical terms and when implemented in the context of teaching practice. The knowledge base, intended to differentiate between knowledge of mathematics as a subject discipline (procedural meaning) and knowledge about mathematics and its application in teaching and learning (what is created through conceptual meaning) (Goulding, Rowland, & Barber, 2002). The knowledge base consists of two primary parts, definition and significance, and two kinds of knowledge, substantive (SUB) and syntactic (SYN). In theory, comparing these parts provides a conceptual picture of how mathematics should be known and how it should be taught (Askew, 2020). SUB and SYN knowledge are important components of pre-service teachers' mathematics subject matter knowledge, in conjunction with theoretical-pedagogical design, i.e., their pedagogical content knowledge (Shulman, 1987; Zaslavsky, 2008; Venkat & Askew, 2018). Researchers argue that teaching mathematics is not an easy undertaking, and that preparing pre-service teachers to teach mathematics is thus a complex process that requires ample forethought and preparation. The development of pre-service teachers' professional knowledge of teaching mathematics is tied to their knowledge of fundamental mathematics and its application via explorative classroom-based. In terms of SUB and SYN contexts, having a knowledge base means that pre-service teachers should learn general principles for the construction of subject mathematics, i.e., "knowing that," and how they can organize teaching mathematics content, i.e., "knowing how" (Turner-Bisset, 1999). In a substantive sense, mathematics focuses on mathematical facts as mathematical structures, concepts and methods, their relationship within the subject of mathematics, as well as essential properties and the relationships between facts, mathematical structures and concepts in the context of conceptual mathematics and its application in counting. Insofar as it calls for epistemic knowledge, syntactic knowledge is about how mathematical structures and concepts originate from mathematical ideas. It has developed through history and in different cultural contexts. SYN knowledge provides the foundation for knowledge about what and how to teach and for developing of teaching experiences in progress (Anderson & Clark, 2012).

This paper examines what pre-service teachers should learn in order to effectively teach mathematics, using subject matter knowledge as a theoretical starting point and through the extension of SUB and SYN structures.

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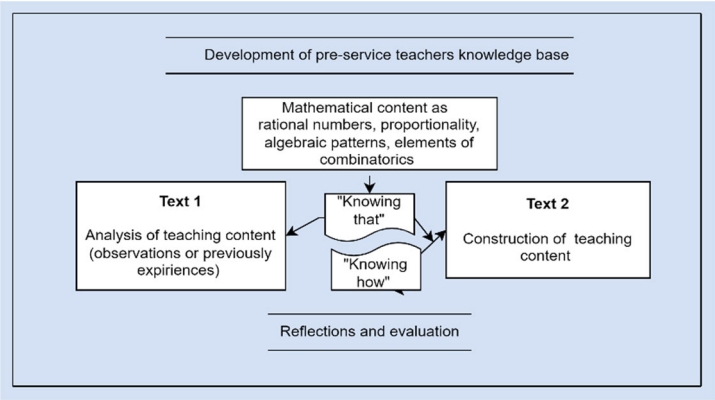
3. Methodology

3.1. Context and participants

The context of this study is a substantive and syntactic component concerning elementary school mathematics education for the K-3 program course *Advanced Math Development,* and the Grades 4-6 program course *Mathematics 4* (both bachelors' level programs). The study focuses on the student's individual work, such written analysis of observations regarding, e.g., teaching content and their own construction of a teaching moment. To track the development of pre-service teachers' mathematics knowledge base, the following conceptual chart is used (Figure 1).

Figure 1. Context for the study.



Both courses were conducted during Semester 6 of two elementary school teacher education programs, in the 2023 academic year. In both programs, most of the participants were female. Both courses had a length of five weeks and participants had no other courses during this time.

K-3 pre-service teachers had four days of practicum work involving observations and analysis of teaching content. After the practicum, the pre-service teachers were given two weeks to produce a written analysis of their observations. At the end of the course, the pre-service teachers presented their detailed plan for a teaching moment with mathematics–specific content. The crucial mathematical content from which to choose were algebraic patterns, rational numbers, proportionality, or basic combinatorics. For their second assignment, the pre-service teachers were asked to plan mathematical content with the same mathematical content as in the proposal for Text 1. The mathematical content for Texts 1 and 2 did not need to be the same.

The Grades 4-6 program course did not include a special practicum component. Instead, the student teachers used their experiences from observations during an earlier practicum (part of a school-based training course). At the end of the course, the pre-service teachers were asked to produce two written analyses (Text 1 and Text 2); see Figure 1. They could choose from the same crucial mathematical content as in the K-3 course. Pre-service teachers from the K-3 and 4-6 programs had different initial learning conditions, as well as direct access to practicum work and different timeframes for producing Text 1. The observation of fundamental differences and similarities concerning these different conditions was especially emphasized. The important part of the study entailed analyzing pre-service teachers' development of substantive and syntactic knowledge, as well as the relationship between them (see Figure 1).

3.2. Data collection and analysis

Data include anonymized copies of individual students' Text 1 and 2 assignments; twenty pre-service teachers' assignments from the K-3 teacher education program and twenty-five pre-service teachers' assignments from the Grades 4-6 program (Corbin & Strauss, 2008). These data were analyzed using a conceptual knowledge base comprising substantive and syntactic components, in order to examine how the pre-service teachers described knowledge in the substantive sense of "knowing that," i.e., mathematical facts and concepts (Text 1), and in the syntactical sense of "knowing how," i.e., the systematic analysis and evaluation of concepts in terms of teaching, argumentation and logical ability (Text 2). The first component addressed pre-service teachers' analysis of teaching content, and the second component addressed their construction of a teaching moment. Data were interpreted from the pre-service teachers' written texts.

4. Results

4.1. Findings: Substantive knowledge and Text 1

In total, clear elements of a substantive component were identified in twelve K-3 pre-service teachers' and ten Grades 4-6 pre-service teachers' assignments (Text 1). Among these, it was obvious that in the analytic process it was difficult for both K-3 and Grades 4-6 pre-service teachers to distinguish conceptual facts from teaching and facts related to subject mathematics, as well to organize a sequential description and analysis of concepts (SUB component). Even if observed teaching included elements of combinatorics or algebraic patterns, the written analysis focused on the limited, fact-based conceptual space, mostly on natural numbers and counting with natural numbers. These results were found both in the K-3 and the Grades 4-6 assignments (Text 1). It was obvious that it was difficult for pre-service teachers to relate their own knowledge of subject mathematics (SUB component) to teaching practice. Instead, they used the content they knew best, such as natural numbers and other easily understood content. The pre-service teachers had different timeframes in which to reproduce Text 1, but those differences had no evident impact on the quality of their texts. The K-3 pre-service teachers had two weeks to produce Text 1 (based on direct observations) and the Grades 4-6 pre-service teachers had five weeks to work on Text 1. Both groups were found to face the same challenges in expressing the substantive sense of knowledge.

4.2. Findings: Syntactic knowledge and Text 2

The analysis of Text 2, which consisted of pre-service teachers' detailed plan for a teaching moment, indicated a positive change in their achievement of SUB knowledge. Eighteen of twenty K-3 pre-service teachers and sixteen of twenty-five Grades 4-6 pre-service teachers were able to provide a theoretical account of a SUB component and showed signs of efforts to overlap SUB and SYN components in their texts. During the course, most of the student teachers made a conceptual development towards transformative SUB-based thinking and were able to interpret this in the content of their teaching moment. But it was still a challenge to systematically assimilate the content in a teaching situation. It was difficult for them to change perspective, i.e., to change the perspective of the analysis from a student perspective to that of a teacher and take on the role of a teacher. This led to the statement of mathematical facts, rather than the construction of a teaching moment. These findings indicate that subject-specific content knowledge is increasingly important to pre-service teachers' ability to construct syntactic knowledge.

5. Conclusion

The purpose of the study was to analyze pre-service teachers' knowledge base and its development in the context of substantive subject matter knowledge and the concept of syntactic subject matter knowledge. Some interesting findings emerged from this study. Firstly, mathematical content such as algebraic patterns, rational numbers and elements of combinatorics requires a certain conceptual level of knowledge on the part of pre-service teachers in order to be assimilated. Pre-service teachers' performance was weaker when it came to algebraic patterns and rational numbers. They tended to stick with the mathematical content they remembered from lectures and seminars in their analysis, rather than using it combinatorically. In short, they were able to imitate but not to create something of their own. At the same time, combinatorics was the best-described learning field for pre-service teachers. Our hypothesis is that they learnt combinatorics via experimentation (visualization), whereas algebraic patterns and rational numbers are more abstract forms of mathematics. This finding also indicates that pre-service teachers' knowledge of mathematics and how they learnt mathematics in school still has a strong impact on their learning with regard to both components but affects the SYN component much more. The study highlights the importance of paying more attention to how pre-service teachers learn mathematics for teaching. In summation, the results of the study reveal student teachers' challenges in grappling with mathematical concepts, as well in their development of SUB knowledge. This means that in education programs for mathematics teachers, more attention ought to be paid to developing pre-service teachers' knowledge base in the form of SUB knowledge and the transformation from a SUB to SYN thinking structure, as well as the impact of this transformation on student teachers' learning of mathematics for teaching. The study provides a broader understanding of the process by which student teachers learn mathematics for teaching purposes. It was clear that challenges related to the substantive and syntactic components of the knowledge base need further investigation. The study also illustrates that pre-service teachers' learning ability depended on their theoretical knowledge, and that practicum work was not sufficient for developing such a knowledge base. More research is needed to clarify an optimal relationship between the development of theoretical knowledge of mathematics for teaching and practicum, with a focus on advantages and disadvantages. This study has limitations that must be taken into account. Therefore, its findings are not generalizable and are tied to the specific context of the study. Still, they provide new knowledge about the preparation of pre-service teachers for teaching mathematics.

References

Anderson, D., & Clark, M. (2012). Development of syntactic subject matter knowledge and pedagogical content knowledge for science by a generalist elementary teacher. *Teachers and Teaching, 18*(3), 315–330.

Askew, M. (2020). Identifying Effective Mathematics Teaching: Some Questions for research. *African Journal of Research in Mathematics, Science and Technology Education, 24*(1), pp. 1–9. Retrieved from https://doi.org/10.1080/18117295.2019.1710049

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching. What makes it special? *Journal of Teacher Education, 59*(5), 389–407.

Boaler, J. (2002). The development of disciplinary relationships: Knowledge, practice, and identity in mathematics classrooms. In A. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education* (Vol 2, pp. 113–120). Norwich, UK: University of East Anglia.

Corbin, J., & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Thousand Oaks, CA: SAGE Publications.

Goulding, M., Rowland, T., & Barber, P. (2002). Does it matter? Primary teacher trainees' subject knowledge in mathematics. *British Educational Research Journal, 28* (5), pp. 689–704.

Darling-Hammond, L., & Bransford, J. (Eds.). (2005). *Preparing teachers for a changing world: What teachers should learn and be able to do*. San Francisco, CA: Jossey-Bass.

Dewey (1998). *Experience and education: The 60th anniversary edition*. West Lafayette: Kappa Delta Pi. (Original work published in 1938)

Hardy, I., Decristan, J., & Klieme, E. (2019). Adaptive teaching in research on learning and instruction. *Journal for Educational Research Online*, *11*(2), 169–191.

Hordern, J., & Tatto, M. T. (2018). Conceptions of Teaching and Educational Knowledge Requirements. *Oxford Review of Education, 44,* 686–701. Retrieved https://doi.org/10.1080/03054985.2018.1438254

Jakobsen, A., Thames, M. H., & Ribeiro, C. M. (2013). Delineating issues related to horizon content knowledge for mathematics teaching. In B. Ubuz, C. Haser, & M. A. Mariotti (Eds.), *Proceedings of CERME 8, the eighth congress of the European society for research in mathematics education* (pp. 3125–3134). Antalia, Turkie: ERME.

Leikin, R., & Zazkis, R. (2010). On the content-dependence of prospective teachers' knowledge: A case of exemplifying definitions. *International Journal of Mathematical Education in Science and Technology, 41*(4), 451–466. Retrieved from https://doi.org/10.1080/00207391003605189

Mezirow, J., & E. Taylor (Eds.). (2009). *Transformative Learning in Practice: Insights from Community, Workplace and Higher Education*. California: Jossey-Bass.

Norton, S. (2019) Middle school mathematics pre-service teachers' content knowledge, confidence and self-efficacy, *Teacher Development, 23*(5), 529–548. Retrieved from https://doi.org/10.1080/13664530.2019.1668840

Shulman, L. (1986). Paradigms and research programmes in the study of teaching: A contemporary perscpective. In M. C. Wittrock (Eds.), *Handbook of Research on Teaching* (pp. 505–526). New York, NY: Macmillan.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harward Educational Review, 57*, pp. 1–22.

Turner-Bisset, R. (1999). The knowledge bases of the expert teacher. *British Educational Research Journal, 25*(1), p. 39–55.

Venkat, H., & Askew, M. (2018). Mediating primary mathematics: Theory, concepts, and a framework for studying practice. *Educational Studies in Mathematics, 97*(2), 71–92.

Zaslavsky, O. (2008). Meeting the challenges of mathematics teacher education through design and use of task that facilitate teacher learning. In B. Jaworski & T. Wood. (Eds.), *The Mathematics Teacher Educator as a Developing Professional. Handbook of Mathematics Teacher Education* (Vol. 4, pp. 93–114). Rotterdam, Netherlands: Sense Publishers.