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UNLOCKING CREATIVITY: KNOTS AND SONA DRAWINGS IN MATHEMATICS TEACHER PROFESSIONAL DEVELOPMENT

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Abstract

Art and mathematics, clearly distinct domains, converge in a fascinating exploration that transcends conventional boundaries. The essence of art lies in producing profound, individualistic responses, inspiring a search for coherence in the visual and emotional tapestry it weaves. Simultaneously, mathematics, conventionally confined to analytical rigor, reveals an unexpected facet as an incentive for creativity. This synergy forms the background for an enriching journey into the interplay of mathematics, art, and ethnomathematics. Within ethnomathematics, the bridge connecting mathematical practices and diverse cultural heritages emerges as a powerful tool for educators to generate a multifaceted understanding of both subjects. The core of a 2023 professional development course for mathematics teachers at the University of Aveiro was the fusion of mathematics, art, and ethnomathematics. Targeting grades 5 to 12, the chosen topics were knots and Sona designs. Knots, essential in everyday life, hold a particular place in mathematics through the theory of knots. The introductory concepts of this theory can be linked to the teaching of several mathematical school topics and provide a powerful tool for 3D visualization and reasoning. Sona designs and geometric sand drawings serve as memory aids in the Chokwe people's oral storytelling tradition in south-central Africa, among other African communities. These drawings have mathematical properties related to symmetry, graphs, and knots. The course unfolded in two phases: an exploration of mathematical concepts and their artistic applications, followed by the conception and execution of a project where participants translated some of these concepts into individual artistic creations, using ceramics for the Sona drawings and textile art for the knots. This study explores mathematics teachers' perceptions of an interdisciplinary approach connecting art and mathematics and its potential integration into teaching practices. The research question focuses on whether incorporating art in mathematics teaching enhances comprehension, facilitates interdisciplinary learning, promotes collaborative teaching experiences, and fosters a more positive attitude toward mathematics among students. Employing a mixed case study design, the research combines quantitative and qualitative methods, including direct observation, document collection, a final questionnaire, and field notes for a comprehensive data analysis. Findings are presented through artworks and analysis of questionnaire responses and field notes. In conclusion, the study underscores that infusing artistic contexts within an interdisciplinary framework strengthens and reshapes mathematical concepts. This approach establishes meaningful connections across diverse knowledge domains, nurtures collaborative and engaging teaching and learning environments, and facilitates the design of classroom activities fostering a positive attitude toward mathematics.

Keywords: *Mathematics education, interdisciplinarity, art, ethnomathematics, knots, Sona drawings, professional development.*

1. Introduction

Elliot Eisner (2004), a pioneer in art education, proposed that adopting an artistic approach to education can improve its quality and lead to a new vision of teaching and learning. Given his preference for aesthetic forms of knowledge and learning, he would certainly agree that the arts can contribute to developing more engaging, stimulating, and challenging mathematical tasks.

The connection between Mathematics and art is known since ancient times, and gained greater attention in recent years (Breda, Carvalho & Hall, 2023). This synergy may be further enriched by joining ethnomathematics. Connecting mathematical practices with diverse cultural heritages emerges as a

powerful tool for educators to generate a multifaceted understanding of both subjects (D'Ambrósio & Rosa, 2016). The authors of this work believe that using this tool promotes contextualized learning, which will help to engage and motivate students in learning mathematics. In this sense, and adopting a STEAM approach, the authors of this work organized a professional development course for mathematics teachers at the University of Aveiro, in which connections are established between mathematics, art, and ethnomathematics. The chosen topics were knots and Sona designs.

The theory of knots dates back to the second half of the 19th century when Kelvin attempted to interpret the properties of atoms in terms of knots in the ether. According to Kelvin's theory, each chemical element would correspond to a particular knot. Although Kelvin's model proved to be inadequate from a chemical point of view, it did lead to the emergence of a new area of mathematics as several mathematicians began to explore the properties of knots and list all possible knots. It is possible to intertwine two or more knots to form a link and there are several notations to identify knots and links. The most widely used, even today, was proposed by James Alexander and Garland Briggs in 1927 and is based on the number of crossings: a link X_n^m has m knots, X crossings, and occupies position n in the tables of knots or links. When $m = 1$, that is, if it is a single knot, the exponent is omitted.

Much of knot theory is concerned with telling which knot diagrams represent the same knot and which do not. Transforming one diagram into another without altering the knot (or transforming the real configurations) may be very difficult. These transformations provide a powerful tool for 3D visualization and reasoning. Some properties of knots are closely related to school topics. For instance, the addition of knots shares several properties with integer multiplication. It can be used as a practical hands-on illustration and application of abstract concepts.

The Chokwe people of south-central Africa and other people of East of Angola and neighbouring areas of Zambia and Congo have a tradition of storytelling using geometric drawings in the sand, called Sona, as memory aids. Typically, these drawings would be traced in the sand winding around a rectangular grid of dots using a finger, while telling the tale. Ideally, the entire figure would be traced without having to remove the finger from the sand so that the entire diagram is constructed with a single curve that does not retrace itself (monolinear design). Additionally, Sona designs are usually such that each region contains exactly one point (perfect designs) and present some type of symmetry. Often the figure could be drawn by following a simple geometric algorithm on a grid of points, such as the one described by Liu and Toussant (2008).

Sona drawings have mathematical properties related to symmetry, arithmetic, computational thinking, graphs, and knots. Therefore, it is not surprising that in the last decades they have been used in schools as a means to teach and promote mathematics, ethnomathematics and anthropology (Veloso, 2022).

2. Methodology

2.1. Methodological options

This study explores mathematics teachers' perceptions of an interdisciplinary approach connecting art and mathematics and its potential integration into teaching practices. The research question focuses on whether incorporating art in mathematics teaching enhances comprehension, facilitates interdisciplinary learning, promotes collaborative teaching experiences, and fosters a more positive attitude toward mathematics among students.

In this perspective, a mixed case study was developed (quantitative and qualitative, based on a logic of complementarity), grounded on a pragmatic paradigm and case study design (Yin, 1994). The study was undertaken in a Portuguese higher education institution involving the participants of a professional development course for in-service mathematics teachers. The teachers responsible for the course in consideration are simultaneously the researchers of this study. In order to develop this experience, the techniques of inquiry, direct observation, and documental collection were applied, and the following instruments were used: final questionnaire and field notes.

2.2. Description of the study

In the professional development course discussed in this paper, our aim was to incorporate all activities into the participants' school routines while providing an opportunity for both learning and a deeply satisfying experience, as previously done in other courses mentioned by Breda et al. (2023). Following the ideas presented by Borko, Jacobs and Koellner (2010), these courses are designed as "opportunities grounded in a conception of learning to teach as a lifelong endeavor", meant to be enjoyable and rewarding. Simultaneously, in this course, we opted for an artistic approach that aligns with Eisner's perspective of a practice rooted in the arts (Eisner, 2004). Teachers take on the role of an artist: they are given time to explore, to create and to surprise themselves.

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The professional development course addressed in this paper was titled “Nós e a Matemática” (“Mathematics and Knots” or “We and Mathematics” given the double meaning of the Portuguese word *nós*). It took place at the University of Aveiro, in 2023, from January 7th to February 4th. Like all professional development courses for Portuguese teachers, it was recognized by the national scientific and pedagogical council for teachers’ professional development (Conselho Científico-Pedagógico da Formação Contínua), being registered with the number CCPFC/ACC - 116242/22. This course lasted for 25 hours; had 13 participants who were mathematics teachers from grades 5 to 12. Given the range of grades taught and the specificity of each level of teaching, different activities were proposed for teachers of different levels (grades 5-6 and grades 7-12).

The course alternated sessions of mathematical topics and applied crafts/arts. In the first session (3h30 face-to-face), the course trainers introduced knot theory and Sona drawings. The topics were explored from a mathematical point of view and complemented with artistic applications. The trainees were given some tasks to apply the concepts. The second session (4h00 face-to-face) was dedicated to the elaboration of Sona drawings on ceramic plates, with the collaboration of the ceramist Purificação Barros. In the third session (3h00 face-to-face) the trainers deepened knot theory and Sona drawing concepts. The 4th session (2h30 face-to-face) was dedicated to further ceramic applications of knots and Sona drawings. In the 5th session (3h30 asynchronous) the trainees had to do some group work and in the 6th session (1h30 online) participants had group discussion about the ongoing homework. In the 7th session (4h00 face-to-face) the participants had to create artistic knots or links using textile materials and the last lesion (4h00 face-to-face) was dedicated to group presentation of all the work undertaken by the participants during the course.

3. Results

3.1. Participants’ artistic outcomes

All participants were asked to make at least two artistic applications: one using knots/links and another using Sona drawings. In this section, we present a selection of the outcomes.

Figure 1 shows three knots made of textile materials. The first is the trefoil knot, 3_1, the second is the cinquefoil, 5_1 and the third is the Whitehead link, 5_1^2.

Figure 1. Textile knots by Sandra Rocha, Ana Deolinda Silva e Teresa Mena.

Figure 2 shows three Sona drawings made on ceramic plates. The first is a 3-linear drawing (three intersecting closed lines) and represents Tshihongo’s mask. The second is 2-linear and represents a human couple (Sachituco and Nachituco). The third shows the construction steps of a particular monolinear design. It represents a plant from which poison is extracted.

Figure 2. Sona drawings on ceramics by Graça Estima, Ester Lemos and Olga Almeida.

3.2. Other results

In this section we analyse of the answers to the questionnaire delivered at the end of the course. The questionnaire had three sections: a first section concerning the characterization of the respondent; a

second section concerning a general evaluation of the course; a third section concerning the use of an artistic approach during the course.

Section 1: Participant characterization

The questionnaire was answered by all the 13 participants. Two respondents taught 5th and 6th grades and the remaining 11 taught grades 7 to 12. Teachers' ages ranged from 43 to 61, with an average of 51.6 and a median of 51 years. Regarding their teaching practice, the participants were mostly very experienced: one participant had between 10 and 15 years of teaching practice and all the others over 20.

Section 2: General evaluation of the course

Teachers were asked to rate, through a five-point Likert scale (1-very negative; 5-very positive), their general evaluation of the course. As can be seen from the heatmap of responses and corresponding statistics in Figure 3, the course was very well evaluated and contributed to increase teacher's knowledge.

Figure 3. Heatmap of responses to the general aspects section; mean and standard deviation.

Question	1	2	3	4	5	mean	std.dev.
What is your global evaluation of the course?	0	0	0	1	12	4,9	0,3
The topics of the course were relevant.	0	0	0	1	12	4,9	0,3
The topics of the course met my expectations.	0	0	0	2	11	4,8	0,4
The activities were interesting.	0	0	0	0	13	5	0
The course contributed to consolidate previous mathematical knowledge.	0	0	1	2	10	4,7	0,6
The course contributed to increase my mathematical knowledge.	0	0	0	0	13	5	0
The course contributed to improve my teaching practice.	0	0	0	1	12	4,9	0,3

Participants were also asked to name three strengths and three weaknesses of the course. We highlight the following strengths: Creativity; New and interesting ideas to apply in the classroom; Broadening of mathematical knowledge and its history in the world; Develops the possibility of interdisciplinarity in the school context.

Section 3: An artistic approach to the teaching of mathematics

Regarding the artistic approach followed during the course, teachers were asked to answer the following Yes/No question: "*Did you like having the opportunity to use an artistic approach to explore mathematical concepts?*". Nine teachers (69%) replied "Yes" and four teachers did not answer. There were no negative answers.

Teachers were also asked to rate their agreement, on a five-point Likert scale (1-total disagreement; 5-total agreement), over a set of statements. As can be seen in Figure 4, the use of an artistic approach in the course was very well evaluated and stimulated the emergence of collaborative/interdisciplinary teaching experiences.

Figure 4. Heatmap of responses concerning the use of an artistic approach in the course; mean and std. dev.

The use of an artistic approach during the course…	1	2	3	4	5	mean	std.dev
contributed to a more positive view of mathematics.	0	0	0	0	13	5	0
made learning more active.	0	0	0	0	13	5	0
contributed to a better understanding, transformation and application of mathematical concepts to diversified contexts.	0	0	0	1	12	4,9	0,3
stimulated the emergence of collaborative teaching experiences, involving teachers and students from different areas.	0	0	0	1	12	4,9	0,3

Continuing the focus on interdisciplinary, teachers were asked to answer, through a five-point Likert scale (1-not at all; 5-very much), to the question "*Do you find it interesting to use an artistic approach in the teaching of mathematics?"* All participants replied "very much".

Finally, teachers were asked to give their opinion (open answer) to the question *"What opinion do you have regarding an artistic approach in the teaching/learning process of Mathematics?*" All ten responses revealed a positive opinion (three didn't answer). Almost all teachers mentioned the importance of this type of approach as a means to increase students' motivation.

Teacher field notes and direct observation confirmed high participant engagement throughout the professional development course. Their curiosity and enthusiasm led to a strong understanding of the

concepts, diverse applications of mathematics, and collaborative teaching experiences. Concrete objectives – exploring videos to use in the classroom and creating individual art pieces – fuelled their motivation to complete all tasks. Notably, even participants without prior artistic experience became increasingly enthusiastic about planning and executing their own projects.

4. Conclusions

The conclusions drawn from this study align with the findings of previous research such as Breda et al. (2023), Breda, Carvalho and Hall (2022), Hall and Pais (2021), Hall, Brás and Pais (2019), and Hall and Pais (2018). These studies all support the effectiveness of an artistic approach in maths education. The current study reinforces this notion by demonstrating the successful integration of artistic and cultural elements into the teaching of mathematics. Positive evaluations received through questionnaires completed by participants at the end of the course, combined with their enthusiastic feedback, provide strong evidence that the course achieved its intended goals. The teachers who participated in the study have enhanced their abilities to design interdisciplinary tasks and projects. The employed methodology has proven effective in fostering a more constructive outlook towards mathematics and increasing motivation among learners. Most importantly, participants have demonstrated a notable degree of enthusiasm and joy, essential ingredients for fostering effective teaching and learning environments.

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