https://doi.org/10.36315/2024v1end043
Education and New Developments 2024

GAMES AND CALCULUS

Maria Paula Oliveira
Department of Mathematics/Center for Research and Development in Mathematics and Applications (CIDMA), University of Aveiro (Portugal)

Abstract

Every 1st semester of an academic year we have around 500 engineering students enrolled in a Calculus 1 course. Involving students in the learning process is a concern for any teacher, thus making us think of strategies to achieve this goal. This academic year we decided to introduce board games in some classes, to encourage mathematical communication and teamwork. In addition, the four weekly contact hours were divided into blocks, and one hour each week was dedicated to solving group tasks which, at the end of the lesson, were submitted on the Moodle platform and made available to all the students in the class. Each group had a different task, so that a wide range of solutions was available to everyone. The performance in these tasks was considered in the course's final grade. Moodle's peer assessment tool was also used, making students actively participate in the curricular unit's assessment process. The groups were randomly created (each student participated in three different groups along the semester) and the students evaluated their performance, as well as their peers' contribution in the group work, twice. Although the percentage of successful students did not improve significantly compared to previous years, the number of students who underwent assessment increased considerably and the attendance rate was also higher. Our university has a quality assurance system and, at the end of the semester, the students evaluate the courses they have taken and the teachers who have taught them. In their perspective, this methodology was useful, with minor corrections. In this paper we'll illustrate some of the games used, describe in more detail how the course worked and reflect on the learning results obtained.

Keywords: *Calculus, games, peer assessment.*

1. Introduction

At the University of Aveiro Calculus I is a compulsory subject for 1st year science and engineering programmes. In our case we have 507 students, from eight different degrees, spread over 10 classes and 5 teachers. As (Iannella, Morando, & Spreafico, 2022) say "The main purpose of general scientific degrees' Calculus courses is to introduce students to the scientific method of analysis, providing a suitable language and useful skills in order to effectively face other disciplinary courses. Unfortunately, this does not happen frequently." Every academic year we have to deal with different profiles and backgrounds of students from a variety of degrees, and often not motivated for the study of mathematics. "Designing actions to promote motivation and increase students' engagement becomes an essential challenge" (Morando & Torconi, 2022) for any teacher.

(Vankúš, 2021) recalls that "The idea of games as an educational tool is not a new one, it was originally devised by Hellenic philosophers, Plato and Aristotle", and he also states that "In the area of mathematics, educational games were identified as suitable to promote mathematic achievements in various domains".

After many years teaching Calculus using different strategies to engage students on their learning process, namely, flipped classroom and team-based learning, I decided to use, in some classes along the semester, board games, following the suggestions of P. Morando and G. Torconi (2022) and using the experience I had in a workshop on Mathematics Games in Vercelli in 2023.

2. Methods

Calculus I has 4 weekly contact hours, in 2 blocks of 2 hours each, and an extra hour which students can use to clear some doubts arose on their study, although they don't use them seldom, except when there is an exam in that particular week. In the 1st semester 2023/24, each block was divided in two different

moments: in the first block some time was used to clarify doubts from the previous classes and the rest of the time was a traditional class, exposing the concepts, giving examples and solving one or two exercises, discussing with the students the possible ways to solve them; the other block started with the traditional class, leaving one hour for the students to work in groups solving different tasks assigned to each group. It was in this last block that some games, involving the concepts taught previously, were played.

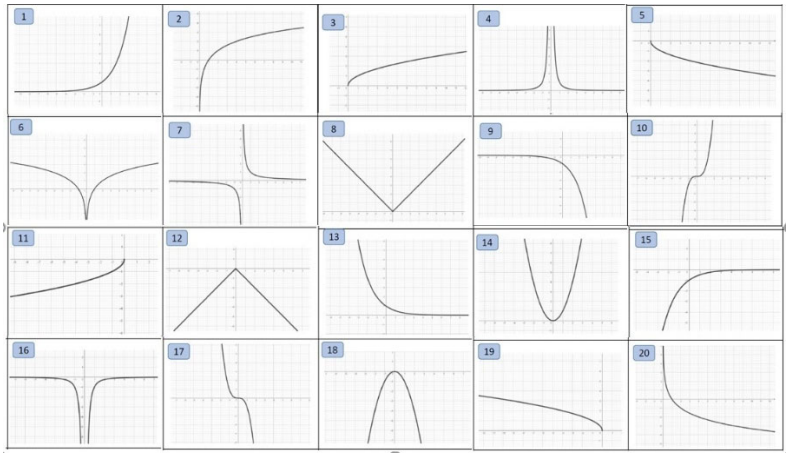
The groups were randomly formed, using Moodle, and they changed group three times. This procedure was used because I believe it is important that students get used to working with different colleagues, but also, as they had to assess their own performance, as well as their peers', the grades were not biased, as each student would be assessed by different classmates.

2.1. The games

The classes started with the game "Guess who" played with the whole class. I chose one graph, amongst 20 that were projected (see Figure 1), and the students had to guess the graph chosen asking questions of yes/no answer. The first student to guess won the game, but if it wasn't the correct choice, the student was out of the game.

This game acted as an icebreaker, allowing the whole class to interact and, at the same time, to remember some of the fundamental concepts for a Calculus course.

Figure 1. Graphs used in the game "Guess who" and in "The mysterious rule".



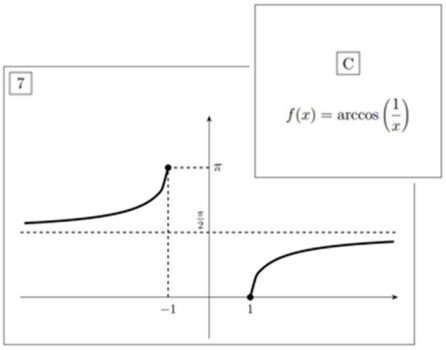
These same graphs were used in the second part of the class in the game "The mysterious rule". In each table there were two teams (of 6/7 students each) that received 2 decks of 20 cards by team, one with characteristics, like $f(x)$ doesn't have points in the second quadrant, or the range of f is a subset of $[0,+\infty[$, and the other with the 20 graphs presented in Figure 1, and also a board like the one in Figure 1.

The characteristics cards are shuffled and each team draws a card. Within 3 minutes they have to pick all the functions that satisfy the characteristic withdrawn. After this the teams exchange the cards with the graphics that fulfil the characteristic and the opposing teams have 3 minutes to guess the characteristic (the mysterious rule). It is important to remark that no graph can be left outside the set satisfying the characteristic, because if that happens and the opposing team realizes that there is one or more functions missing, the team loses one point. The points are distributed like this: if the team chose all the graphs satisfying the rule they get 1 point, if the opposing team finds the rule gets 1 point, but if they find another characteristic that is satisfied only by the graphs selected, among the 20 available, they get 2 points.

Watching the students warmly defend their points of view was very rewarding. And like any good game, at the end there were prizes for the winning teams (chocolates).

Another game used in the study of inverse trigonometric functions was "Math Twins". Each team received a deck with 24 cards: 12 with a graph numbered 1, 2,…12 and 12 labelled A, B, …,M with the analytical expression of a function (see figure 2). They had 10 minutes to form corresponding pairs (graph number, label analytical expression) and write down the pairs in a piece of paper they handed to an opposing team to correct. For each correct pair the team has +1 point and for each wrong pair -1 point. The winner is the team that got more points. The functions chosen were composed of inverse trigonometric functions, so the students had to take into account the domains and range of the composite, which is usually difficult for them.

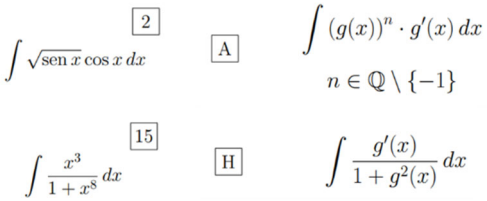
Figure 2. Cards used in the game "Math Twins".



After the game, each team had to do the complete study of one of the functions used to play and submit the detailed resolution in Moodle.

The last game used was "The antiderivatives boxes". Each team was given a set of 25 colored cards (a color for each team) with antiderivatives and 8 "rule" boxes were placed on a table in the middle of the room. The aim of the game was to put the cards with the antiderivatives in the right rule box. For instance, in figure 3, the antiderivative 2 should be put in box A and the antiderivative 15 should go to box H.

Figure 3. Examples used in "The antiderivatives boxes".



Students feel confused when starting the study of antiderivatives, specially on understanding the rules, and playing this game made them discuss with their classmates which rule to use. After the game, each team and a set of 3 antiderivatives to evaluate, and that was the task they had to submit on that class.

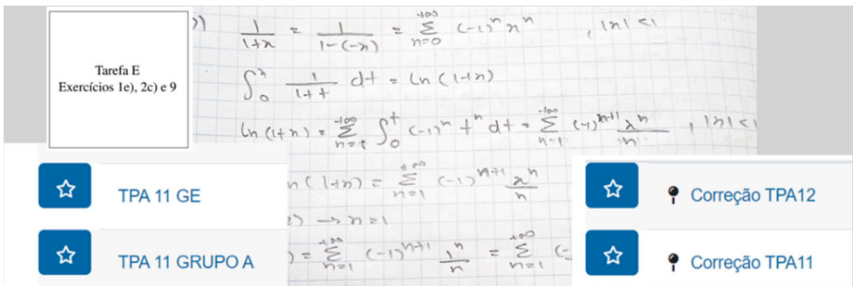
The games used within lectures improve the participation and motivation of students, and develop communication skills in mathematics, as well as promoting the use of mathematics language, fundamental to succeed in a Calculus course.

2.2. Team work

The tasks were proposed in the second part of the second block, after introducing the related topics, illustrated with some examples. Our goal was to have students to immediately reflect on what they have learnt in the last two lessons through peer discussion. Students feel more comfortable when sharing their doubts in a small group rather in front of all class and discussing how to solve a task is important to consolidate learning.

Each group picked a task from a *hat* (I used it to put the sheets of paper with the exercises to be solved) and had to solve it during the remaining time of the class and submit the detailed solution in a Forum available for all the students of the class (see figure 4). I corrected the tasks and gave the feedback before the next class, in order to discuss the difficulties that had arisen when solving them.

Figure 4. The submission and correction of the task.



The first groups were created in the first lesson to play the “Mysterious rule”, but in the beginning of the semester there are always class changes, new students who are placed in the second and third stages of higher education applications, so peer assessment was not applied on these groups. Using Moodle, random groups were created twice along the semester, and it was applied the peer assessment tool so that each student could evaluate their performance and that of their peers, and there was a penalty if a student didn’t answer to the peer assessment quiz. In a scale of 0 to 4, the students had to evaluate their peers in the following items: attendance to classes, relevance of ideas given for group work, quality and quantity of the work done, fulfilment of assigned tasks, contribution to the organisation, cohesion and progression of the group work, ability to listen to other's ideas, encouragement given to others to participate in the work. Although not all the students used this tool responsibly, evaluating all their colleagues with the highest mark - even those who contributed little to teamwork, many were conscious even in their self-evaluation.

3. Discussion

The pass rate hasn’t improved when compared to previous academic years, however class attendance was higher along all the semester (usually, there is a drop in the attendance after the first test), and students were active, specially in the teamwork block.

This methodology had several goals to hit. One of them is a common request that students ask: having solved exercises. Passing responsibility for this request on to the students themselves should force them to commit to their learning, providing detailed and carefully written solutions to their peers. Although this wasn't always achieved, I believe it was worth it, and the students recognized this fact.

The university has an internal quality assurance system (IQAS) established and the students are invited to answer a survey regarding their self-assessment, the functioning of the course and their teacher. Overall, the students are not particularly motivated by the Calculus I course nor satisfied with their own performance, with an average of 5.89 and a median of 6, on a scale of 1 to 9. Regarding the functioning of the course the students consider that it is appropriate, as well as the study materials provided and the assessment method used. The average teacher evaluation was 7.85 in 9, with 41% of the students grading it with the maximum grade. In the reports produced as part of the IQAS, group work carried out in class is mentioned as an added value because it generates cooperation between peers and facilitates learning.

Using these strategies, allowed me, as a teacher, to perceive the difficulties students had in solving the proposed tasks, but, the large number of students per class (45-50 students) hindered a more personalized attention to all of them.

4. Conclusions

In the 21st century the teacher’s role must adapt to the reality of a global world, where the internet knows it all. It’s important that we endow our students with critical thinking and communication skills, teamwork and self-confidence, and not so much with routine procedures that they can easily find on the internet.

Along the years I have been adapting my way of teaching to the audience I had and realized that, even if my effort on solving and explaining in the board was huge, only a few students would profit. I believe that when they try to solve problems/exercises by themselves, with the help of their peers, is the best way to learn. Working in small groups also encourages students to ask questions to their classmates, they wouldn’t feel at ease in front of the all class. It’s also known that one of most efficient techniques to learn is to teach, and this can happen when students are grouped and they have to explain to the others the strategies used to solve the task they have to do.

The use of games seems to be an appropriate strategy to learn some concepts, and to promote the interaction between students. Despite some dissenting voices, in particular because they take up a lot of class time, the learning results are worth the effort.

Acknowledgments

This work is supported by the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), within the Center for Research and Development in Mathematics and Applications (CIDMA), through project references UIDB/04106/2020 and UIDP/04106/2020.

References

Iannella, A., Morando, P., & Spreafico, M. L. (2022). Challenges in Mathematics Learning at the University: An Activity to Motivate Students and Promote Self-awareness. In G. Casalino, M. Cimitile, P. Ducange, N. Padilla Zea, R. Pecori, P. Picerno, & P. Raviolo (Eds.), *Higher Education Learning Methodologies and Technologies Online*, 1542, pp. 321-332. Cham: Springer International Publishing. doi:10.1007/978-3-030-96060-5_23

Morando, P., & Torconi, G. (2022). Brains on in Math classes. *EDULEARN22 Proceedings* (pp. 1522-1529). Palma, Spain: IATED. doi:10.21125/edulearn.2022.0401

Vankúš, P. (2021). Influence of Game-Based Learning in Mathematics Education on Students' Affective Domain: A Systematic Review. *Mathematics, 9*(9). doi:10.3390/math9090986