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CONSIDERING AUTOMATIC FEEDBACK IN ASSESSMENT FOR MATH LEARNING

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Abstract

In this study, we present the partial results of an ongoing project for teacher training in Portuguese-speaking countries, such as Brazil, Portugal and Cape Verde. The main objective is to identify strategies for evaluation processes using resources with automatic feedback, aiming to contribute to the improvement of students' mathematical learning. Approaching and discussing evaluation implies questioning fundamental aspects of the educational environment, seeking adjustments that improve pedagogical procedures, since the nature of evaluation evolves in methods and purposes, requiring an epistemological change to understand students' "mistakes". Some research pointed that in the evaluation process, especially in the context of learning, the interpretation of students' difficulties in mathematics is identified through the analysis of their "errors". In addition, didactic strategies, such as automatic feedback, are proposed in other studies as an attempt to overcome these difficulties. As it is a study focused on evaluation in mathematics education, the choice of the theoretical framework explores two fundamental areas of research. These include assessment of learning, which takes a comprehensive approach, covering both assessment of learning and assessment for learning. In addition, a theoretical framework is considered, in which assessment in mathematics education is recognized as an essential component in the construction of mathematical knowledge, especially regarding the treatment of errors during the educational process. We believe that the integration of technological resources into the assessment process can contribute significantly to mathematics learning. The involvement of aspects such as automatic feedback makes it possible to make decisions to achieve the objectives proposed in the development of the content. So far, six meetings have been held that indicate, in general, a positive perception of the participants in relation to the potential of automatic feedback in teaching and assessment in Mathematics. They highlight its practical applicability in the development of cognitive skills and in the treatment of errors. Additionally, if teachers have a minimum knowledge to reparametrize the applications built, they can refine the automatic feedback process, incorporating the acquired knowledge into their practice.

Keywords: *Mathematics education, evaluation, automatic feedback, teacher training.*

1. Introduction

This article presents partial results of an ongoing project aimed at teacher training in Portuguese-speaking countries such as Brazil, Portugal, and Cape Verde. The main objective of the project is to identify strategies for assessment processes using resources with automatic feedback, aiming to contribute to the improvement of students' mathematical learning.

The authors of this project conduct research and actions in the context of teacher training for the use of digital technologies, particularly with the use of GeoGebra in public schools in Brazil, Portugal, and, with the support of the OEI, in some African countries such as Angola, Mozambique, and Cape Verde (Dos Santos, Silveira, & Trocado, 2020; Abar, 2020; Abar & Rodrigues, 2020).

The analysis of assessment in education and the interpretation of results highlight challenges faced in pedagogical practice to ensure conditions and didactic means that stimulate students in their studies, promoting an environment without intimidation and facilitating the understanding of mistakes made. Addressing and discussing assessment implies questioning fundamental aspects of the educational environment, seeking adjustments that improve pedagogical procedures, since the nature of assessment evolves in methods and purposes, requiring an epistemological shift to understand students' "errors."

We consider that resources for the assessment process, with the use of technologies, can contribute to students' learning in mathematics by involving aspects of automatic feedback, allowing decision-making to achieve the proposed objectives of the content under development.

The development of training, embedded in the concepts presented above, will allow recognizing the teacher's interpretation in the face of possible errors by students, how they intervene, and what feedback they provide to contribute to an assessment for mathematical learning.

2. Framework

Within the scope of mathematics teaching and learning processes, it is important for assessment to consider data regarding the difficulties present in students' activities and provide support for the reorientation of school practices, considering such difficulties as opportunities for improving assessment.

The rapid transformations of today's society demand differentiated assessment strategies, such as automatic feedback and the use of technologies, so that their mechanisms cater to a generation of students who can be protagonists of their own learning. Analysing the errors and successes of an assessment allows the possibility of understanding how students' appropriate knowledge and, thus, allows for the construction of suitable automatic assessment processes (Abar, Dos Santos, & Almeida, 2022).

In the creation of assessment resources with automatic feedback, the indications of authors Buriasco and Soares (2012) can be considered, as they suggest that assessment "should gauge its ability to find patterns, seek regularities, read tables and graphs, relate data, build schemes, and develop procedures." (p. 111).

As this is a project on assessment in the context of mathematics education, the choice of theoretical framework was made to address three areas of research: (i) learning assessment, which will provide insights into assessment broadly, i.e., learning assessment or assessment for learning; (ii) assessment in mathematics education, which constitutes a specific framework for the construction of mathematical knowledge considering error treatment as an integral part of this construction, especially in the context of educational process; (iii) assessment that can provide resources for its creation with automatic feedback.

The understanding of an evaluative process aimed at contributing not only to the improvement of resources but also to the phases leading to mathematical learning is directly linked to the training of the teacher who will conduct this process in school environments.

The foundations of a mediating evaluative action go beyond studies on assessment theories and require in-depth knowledge of knowledge theories, as well as studies related to specific areas of the teacher's work.

For Buriasco and Soares (2012, p. 110), "the assessment of mathematical learning should be seen in schools as a research process, an activity shared by teachers and students, of a systematic, dynamic, and continuous nature". Thus, "all instruments used in the assessment of mathematical learning should be seen as research instruments, which enable capturing how students solve proposed situations." (Buriasco & Soares, 2012, p. 114). The fact that a student makes an error in solving a problem situation or even in an algorithm already explored by the teacher does not mean the absence of all mathematical knowledge necessary to solve the proposed situation. It can be a revealing fact about which knowledge the student failed to mobilize to succeed in the final response, or rather, how far the student managed to use their knowledge in attempting to solve the problem, or even which knowledge was lacking for them to succeed in the task. Faced with this fact, it can be used as a way to understand which mathematical knowledge needs to be developed by the student.

Error can bring clarity to the teacher and to the student themselves in the tasks of teaching and learning mathematics, respectively. The way assessment is conducted by the teacher is strongly associated with how mathematics teaching and learning are conducted by them.

Published works by researchers present considerations on the assessment process, in and for learning, seeking to interpret students' difficulties in mathematics identified through the analysis of "errors". Investigations conducted point to reflections on the assessment process, especially in the context of learning, interpreting students' difficulties in mathematics identified through the analysis of their "errors". In addition, didactic strategies, such as automatic feedback, are proposed in other studies as an attempt to overcome these difficulties.

In her research, Cola (2015) reveals that the "assessment processes that permeate the school environment are quite superficial" (p. 84) and that in the context of mathematics education, assessments incorporate elements of a "traditional evaluative practice that does not privilege the perspective of error" (p. 84).

The analysis of errors and successes in assessment allows understanding how students appropriate knowledge and thus enables the construction of appropriate automatic assessment processes (Abar et al., 2022).

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The role of error becomes fundamental in the assessment process. According to Cury (2007), error should not be seen as something to be avoided at all costs, but rather as a powerful tool for investigation, by both the student and the teacher, to guide the teaching and learning of mathematical knowledge.

As it is a study focused on assessment in mathematics education, the choice of theoretical framework explores two fundamental areas of research. These include learning assessment, which adopts a comprehensive approach, covering both learning assessment and assessment for learning. Additionally, a theoretical framework is considered, in which assessment in mathematics education is recognized as an essential component in the construction of mathematical knowledge, especially concerning the treatment of errors during the educational process.

We believe that the integration of technological resources into the assessment process can significantly contribute to learning in mathematics. Involvement of aspects such as automatic feedback enables decision-making to achieve the proposed objectives in content development.

3. Methods

Teachers from both public and private institutions were invited and encouraged by the authors to join and participate in the project through their respective institutions. Fourteen teachers joined, including one from Mozambique, one from Cape Verde along with three of their students, six from Portugal, and three from Brazil.

The monthly work dynamics, conducted through the Microsoft Teams platform, are guided by active and collaborative participation in practical and theoretical activities. The association between practice and theory is encouraged, as well as the manipulation and analysis of problem situations. Five groups were formed, in which participants are encouraged to deepen their skills in exploring the following freely chosen themes: Calculus and Trigonometry, Music and Computational Thinking, Quadratic Functions, Exponential Functions, and Addition of Integers in the Context of Financial Education.

Six meetings have been conducted so far, indicating a generally positive perception among participants regarding the potential of automatic feedback in teaching and assessment in Mathematics. They highlight its practical applicability in developing cognitive skills and in error treatment. Additionally, if teachers have a minimal knowledge to reparametrize the applications built, they can refine the process of automatic feedback, incorporating the acquired knowledge into their practice.

The resources collaboratively created will be used in teaching practice with their students, and experiences will be shared in other work meetings for possible improvements and dissemination.

The involvement of teachers in synchronous meetings was significant, and some activity proposals were conceived, one of which is outlined in this article, characterizing the three research areas already discussed in this text: (i) learning assessment; (ii) assessment in mathematics education; (iii) assessment with automatic feedback.

The proposals are discussed, in each meeting, with the participants, sharing the steps of activity construction. Figures 1 and 2 present one of the activities for feedback on Exponential Functions developed by one of the groups and the respective guidelines.

Figure 1. Game with Automatic Feedback.

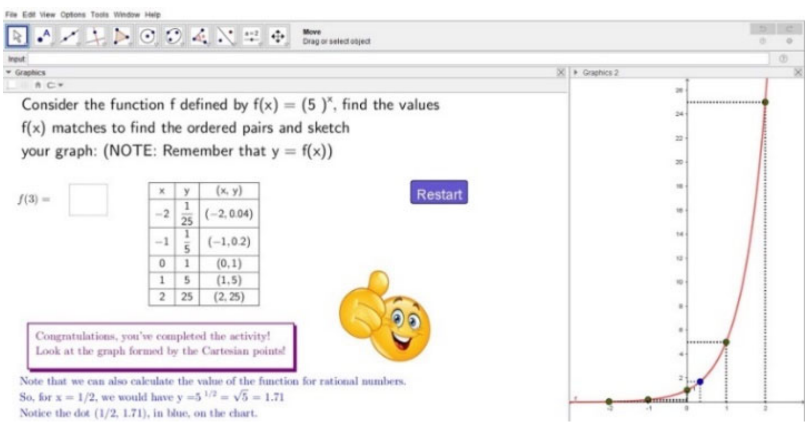

Figure 2. Guidelines for building Feedback.

<table>
<tr><th>Type of errors</th><th>Example</th><th>Feedback</th><th>Condition</th></tr>
<tr><td>Do not invert the fraction with a negative exponent</td><td>$2^{-1} = 2$</td><td>Power property:
$a^{-n} = \frac{1}{a^n}$</td><td>f(n) ≐ 1 / ry ∧ n ≠ 0 ∧ IsSet(ry)</td></tr>
<tr><td>Put result equal to zero (n = 0)</td><td>$2^0 = 0$</td><td>Remember that every number to the power of zero, except zero, is equal to one.</td><td>ry ≐ 0 ∧ n ≐ 0 ∧ IsSet(ry)</td></tr>
<tr><td>Put result equal to one (n = 1)</td><td>$2^1 = 1$</td><td>Remember that every number raised to one is equal to itself.</td><td>ry ≐ 1 ∧ n ≐ 1 ∧ IsSet(ry)</td></tr>
<tr><td>Put negative result
a>0</td><td>$2^{-1} = -2$</td><td>The exponential function of type f(x) = a^{x} assumes only positive real numbers as a result.</td><td>ry < 0 ∧ IsSet(ry)</td></tr>
<tr><td>Multiply base and exponent (n > 1) natural numbers</td><td>$2^3 = 6$
$2^{-3} = -6$ *</td><td>$a^n = \underbrace{a.a.a.\dots.a}_{n\ factors}$
$2^4 = 2.2.2.2 = 16$</td><td>ry ≐ a n ∧ n > 1 ∧ a ≠ 2 ∧ IsSet(ry)</td></tr>
<tr><td>Generic error</td><td></td><td>To find the value of f(n), simply replace x with n in the given function.
Here's an example:
f(x) = 2^{x}, então f(3) = 2^{3} = 2.2.2 = 8.</td><td>f(n) ≠ ry ∧ IsSet(ry) ∧ ¬ ((f(n) ≐ 1 / ry ∧ n ≠ 0) ||(ry ≐ 0 ∧ n ≐ 0)||(ry ≐ 1 ∧ n ≐ 1)||(ry < 0)||(ry ≐ a n ∧ n > 1))</td></tr>
</table>

Following testimonial from one of the members of the Exponential Functions group, regarding the questions posed by the authors, see Table 1, we can observe the understanding of the project proposal.

Table 1. Testimonial from one of the members of the Exponential Functions group.

Question	Participant answers
Reflection on recurring students' errors	
When developing the mathematical topic of your group, did you reflect on the recurring errors that can be identified in student assessment? What errors or difficulties were identified?	"We identified that the main failures could occur due to the improper use of power properties. For example, the student may not pay attention when a power has a negative exponent, confuse the properties when the same number is raised to zero or one, not realize that an exponential function of the form a^x requires a positive codomain, and perform the multiplication of the base by the exponent."
Challenges in develop automatic feedback tasks	
What strategies does your group consider most suitable for overcoming student difficulties? Describe (possible) initiatives adopted to try to overcome them	Yes, it was evidenced that in activities like this, the biggest challenge is to anticipate the possible student failures to give a clue so that he can understand his mistake without giving the answer. Thus, we try to give clear feedback with examples that facilitate the understanding of the content covered."
Auto sufficiency of automatic feedback hypothesis	
Has your group found references in academic articles or research that address similar challenges? If so, how have these sources influenced or enriched your group's proposal?	"Giving specific hints for each possible identified error and examples that help to notice their mistake, as well as a generic tip for a possible error that was not foreseen by the team."

By this way, in this section the authors outline the methodology employed in the research project, including participant recruitment, meeting dynamics, data collection methods, and some of the preliminary findings of one of the trainee's groups.

4. Discussion

In the realm of mathematics teaching and learning, assessment must consider student difficulties and provide insights for reshaping classroom practices, viewing these difficulties as opportunities for improving assessment. As seen in Figure 2, one example among others in this project, participants identify common student misunderstandings and formulate feedback considering them.

Today's rapidly evolving society demands differentiated assessment strategies, including automatic feedback and technology integration, to cater to a generation of students who can take charge of their learning. Analysing successes and mistakes in assessments allows understanding how students appropriate knowledge and helps construct suitable automatic assessment processes (Abar et al., 2022). The example of the GeoGebra Tasks (Figure 1) shows how participants in these groups incorporated their content and didactical knowledge, as seen in Figure 2, to build a scenario for student use. Also, it should be noted the inclusion of feedforward feedback, see the left bottom of Figure 1, given the notice to the calculus of a potency with irrational exponents and a positive base, anticipating students' questions or misconceptions.

The collaborative and participatory nature of our project highlights the significance of teacher professional development in enhancing assessment practices. By engaging teachers from diverse backgrounds and facilitating collaborative discussions, we fostered a supportive community of practice focused on innovation and improvement in mathematics education. This approach not only empowers teachers to implement effective assessment strategies but also promotes knowledge sharing and co-construction of pedagogical and technological knowledge, as evidenced in Table 1 participants answers.

While our study has provided valuable insights into assessment practices with automatic feedback, several avenues for future research warrant exploration. One promising direction is to investigate the long-term impact of automatic feedback on students' learning outcomes and retention of mathematical concepts.

5. Conclusions

Based on the testimonials of the teachers, we observe that by creating an initial categorization associated with a response, this approach can be replicated in similar situations and automated, going beyond formative assessment. The records of the results of the proposals can be integrated into summative assessments, expanding the reach and effectiveness of the assessment process.

With the continuation of the project, we hope that the proposals of the participants will be further refined so that they can be published and made available online, thus contributing to teacher training in the context of assessment with automatic feedback.

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