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THE MATHEMATICS THAT THE SEA CONCEALS - CONNECTIONS FOR THE TEACHING OF MATHEMATICS

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Abstract

Exploring mathematical connections with relevant current issues is an important support for learning in a world where reality and its problems are becoming increasingly global, multidimensional and complex. The first author of this paper has developed, from scratch, a set of 17 worksheets relating mathematics with the sea, driven by the cause of ocean sustainability. The tasks are intended for middle school mathematics, grades 7 to 9, and address all the main topics of the syllabus (algebra, geometry, statistics, computational thinking). Most of the tasks are contextualised in the history and livelihood of Cascais, the Portuguese municipality where the author teaches, focusing on aspects such as marine pollution, fishing, oceanographic exploration, surfing, navigation, lighthouses, among others. During the academic year 2022-23, a didactic experiment was carried out, by assigning some of the tasks to middle school students, collecting data and conducting a survey afterwards. From the results of the didactic experiment, the following aspects stand out: the established connections enabled the students to recognise the relevance of mathematics, thus leading them to understand it is part of reality and not merely an isolated school subject; the students got involved with curiosity and commitment in carrying out the tasks; they detected incorrect resolution strategies and self-corrected them; they developed transversal mathematical skills and deepened other areas of knowledge, going beyond learning specific mathematical contents. The tasks provided the students moments for reflection and the sharing of experiences of active citizenship in favour of the ocean, proving that mathematics classes can contribute to raising awareness of the importance of ocean sustainability and benefit from this connection. This work culminates with the challenge of putting into practice or adapting these tasks in other Portuguese schools and municipalities, thus reinforcing the students' learning process in a country with a maritime dimension such as Portugal.

Keywords: *Middle school mathematics, mathematical connections, sea, ocean sustainability, Cascais.*

1. Introduction

The ocean constitutes 70% of the planet, and its sustainability is currently a global concern. Its protection is a Sustainable Development Goal (SDG 14). It is essential to prevent and reduce marine pollution to address challenges such as climate change and the availability of sufficient, safe, and nutritious food. For this, the cooperation and action of everyone is necessary. Young people are not detached from this struggle; on the contrary, when informed, they easily join the cause and can be potential agents of change. In what way, can mathematics classes contribute to the cause of ocean sustainability? Can the topic of the sea contribute to achieving the learning outcomes of mathematics at school? The search for answers to this dual question guided the work presented in this article.

1.1. Mathematical connections in the curriculum

As Edgar Morin emphasises in the epistemology of complexity (Morin, 2002, pp. 39-50), to address the complex and global problems that the current world faces, compartmentalised knowledge is insufficient; it is necessary to reconnect the various areas of knowledge, recognising the relationship of the part to the whole and the whole to the part.

The *Aprendizagens Essenciais de Matemática* (AEM), Portuguese official mathematics syllabus and guidelines, approved in 2021 for years 1 to 9, acknowledge the importance of the ability to establish internal and external mathematical connections (Ministry of Education, 2021a, pp. 17-18). This mathematical ability is considered cross-cutting and should be addressed in an integrated manner with other subjects.

The relevance of external mathematical connections in learning is grounded in the "principles of relevant knowledge" advocated by Edgar Morin (Morin, 2002, pp. 39-50). In his opinion, for the construction of relevant knowledge, education must make evident the context, the global perspective, the multidimensionality, and the complexity of knowledge.

The purpose of mathematical connections is to support learning, facilitating the understanding and expansion of ideas, concepts, representations, and problem-solving strategies in specific contexts. Consequently, it presents mathematics as a cohesive whole, articulated and related to other areas of knowledge. Additionally, it is essential to consider the potential that the context of school tasks has in motivating students. When students are able to establish mathematical connections, their understanding becomes deeper and more enduring (National Council of Teachers of Mathematics, 2008, p. 71).

1.2. The sea as a motivating theme for establishing connections in mathematics classes

With the aim of engaging students and promoting their learning, based on the curriculum guidelines of the Ministry of Education (ME) and inspired by the SDG 14, the first author of this article developed seventeen tasks for mathematics classes from scratch. These tasks establish connections between the sea and the learning objectives of the subject. They are intended for students in the 7th, 8th, and 9th years of basic education, covering various topics defined in the AEM (Ministry of Education, 2021a, 2021b, 2021c). The goal was to diversify the types of exercises, ranging from direct and simple questions to others with a higher level of cognitive demand, including problem-solving and geometric constructions.

Contextualised mainly in the history and characteristics of Cascais, a coastal town near Lisbon, the tasks address topics such as tides, ocean exploration, marine animals, sustainable fishing, surfing, lighthouses, marine pollution, art inspired by the sea, among others. Each task is divided into two sections: the statement for students and a pedagogical framework for teachers. Both parts contain an introductory section with relevant historical/cultural/environmental information about the context. Obtaining all the information required visiting many interesting places (museums, lighthouses, Cascais tide gauge, fish markets, etc.) and interviewing people from many different occupations (fishermen, museum staff, photographers, surfers, etc.) All the problems/questions presented along the task seek to be meaningful within the context and not just a transposition of a mathematics exercise into a forced context. As an illustrative example, one of these tasks is described in section 3. The complete set of tasks is available in Frade (2023).

2. Methodology

This study aimed to analyse how mathematical connections with the topic of the sea, including the issue of ocean sustainability, affect the learning of mathematical content. The case study (Yin, 1994) was the research method used to assess the potential of the created tasks. Three tasks were selected and implemented in 7th-year groups, according to the learning objectives intended to be achieved in each year group at a specific moment.

Inquiry techniques, direct observation, and document analysis were used, employing the following instruments: field notes, questionnaires, and written productions by students. During the task implementation, the teacher (first author of this article) adopted the participant observation technique. Written productions were photographed and sent by students through Google Classroom, and questionnaires were filled out on Google Forms. The questionnaires aimed to understand how the tasks were perceived by the students. The data collected throughout the survey were later analysed, and the results were presented using graphs created with Excel. Stacked bar graphs were constructed according to the five-point Likert scale used in the questionnaires (1 - Total disagreement, 5 - Total agreement), centring and colouring the bars to facilitate the reading.

2.1. Participants

The case study was conducted at a school in the municipality of Cascais and took place between February and June 2023. The participants were students from three 7th-year groups, totalling 85 students.

2.2. Description of the study

The selection of the tasks and the choice of when to apply them were based on the learning objectives planned for the year groups. The first task selected, “Boat speed,” was applied in all groups and addresses functions. The second and third tasks, “The oceanographer king” and “The poem,” were applied in two of the year groups and address scale factors and equations, respectively.

The students worked in pairs, as we believe that this methodology allows students to express their ideas, listen to other opinions, discuss solution strategies, argue and critique, contributing positively to learning. Moreover, this is a methodology regularly used in mathematics classes that is clearly appreciated by the majority of students involved in the study. After the pair work, a collective discussion of each task was conducted, using the projection of students' answers. The objectives were to analyse the different strategies used to answer the same question, correct any errors, systematise the mathematical learnings emerging from the task, and in some situations, discuss the issue of ocean sustainability.

In the next section, as mentioned earlier, we present one of the tasks applied to the students, as well as an analysis of the questionnaire answered by the students at the end of the completion of this task.

3. Results

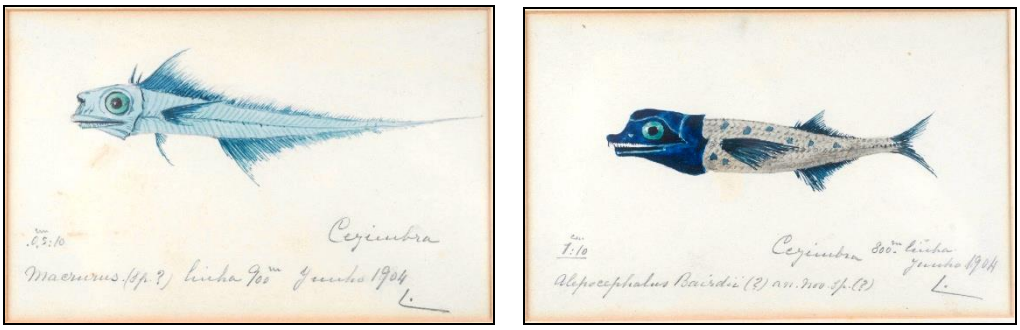
3.1. The oceanographer king

“The oceanographer king” is a task intended for 7th year students and explores the geometry topic of similarity and scale factors. It unfolds around a set of four fish drawings by one of the last Portuguese kings, D. Carlos, who reigned from 1889 until his assassination in 1908 (Portugal became a republic in 1910). He was known as the “diplomat” king due to his multiple diplomatic endeavours, but also as the “father of Portuguese oceanography” due to his interest in oceanography. He is also known for his talent as a painter and among his works there are several scientific illustrations. D. Carlos conducted twelve exploratory campaigns in the Portuguese sea aboard his yachts adapted for oceanographic research and built an oceanographic laboratory to study marine organisms collected during the expeditions (Girard, 1908).

Two of the forementioned scientific fish drawings made by the king have a scale factor indication (see Figure 1). This is a fertile setting for mathematical activities related to the topic of similarity and scale factors. Among the learning objectives for the 7th year related to this topic, one stands out: “identify similar figures in everyday situations,” with the recommended teaching strategy to “confront the meaning of similarity in real life with that of mathematics, in order to provide understanding for students” (Ministry of Education, 2021a, p. 40). “The oceanographer king” complies to this goal connecting mathematics with the areas of scientific illustration and naval modelling. The study of scale factors, more commonly applied in the interpretation of maps, is thus extended to other areas.

The task is enriched by the issue arising from the fact that only photographs of the king's drawings are available, with the scale factor unknown. Having identified and contacted the author of the photographs, Pedro Aboim Borges, it was possible to determine the actual dimensions of the drawings created by D. Carlos. Using this information, it was found that at least two of the fish captured and drawn in 1904 by the king had dimensions larger than those of the same species currently inhabiting the ocean, an aspect proposed to be explored through connections with the field of Biology and fishing sustainability, in particular.

Figure 1. Drawings of two fish by King D. Carlos in 1904, watercolour on paper.
Left: a Macrurus fish, scale factor 0,5:10; Right: a Alepocephalus bairdii fish, scale factor 1:10.



The task contains three parts. In the first one, students are led to discuss the meaning and importance of scale factors and asked to determine some real fish measures using the two drawings of Figure 1. Students are also asked to discuss how can one of the fish lengths to be so much larger than

what is now expected for that species (the fish on the right in Figure 1 measured 1,45m and nowadays this species grows up to 1m.). The second part of the task is related to planning a wooden model of one of the king's yachts, based on a photograph of the yacht and knowing some of its real measurements. Finally, the third part of the task explores the issue arising from the fact that we only have access to photos of the king's drawings without mention of the scale factor. By accessing the photographer's archives, the original photos showed a colour bar next to a ruler in centimetres which allows the determination of the original measures.

3.2. Analysis of questionnaires

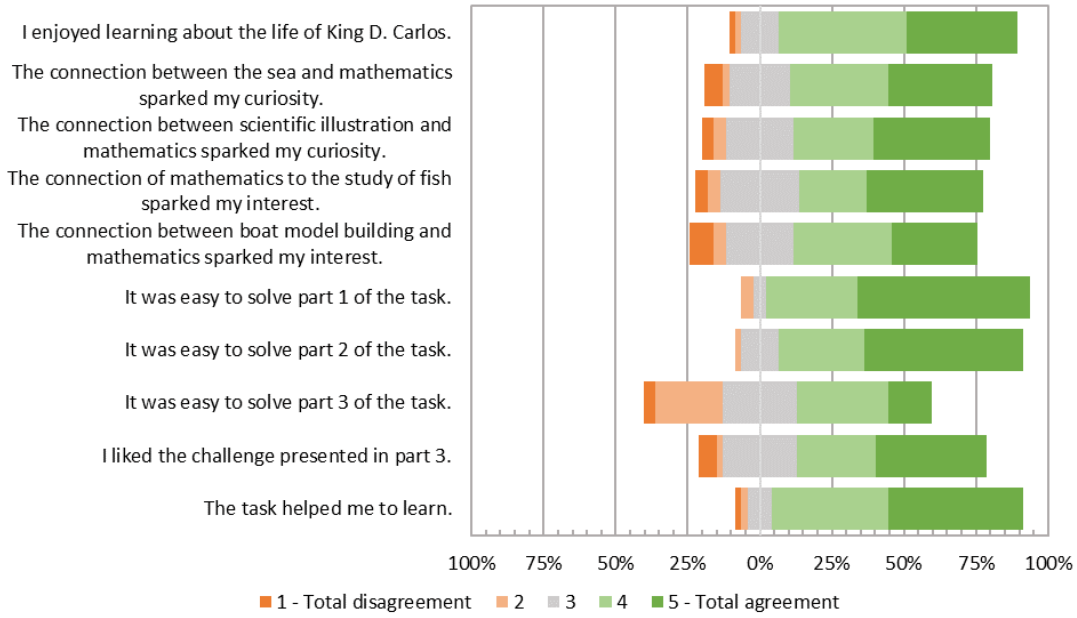
At the beginning of the experiment, students were asked about their interest in the theme of the sea. The answers given by 69 students revealed that 68% of them showed interest, 9% lacked interest, and 23% had a neutral opinion. It should be noted that living in the municipality of Cascais, a seafront a seafront territory, may have influenced the expressed interest.

After the completion of the task "The oceanographer king," students expressed their opinions about it by responding to a questionnaire. Forty-seven responses were obtained.

On a five-point Likert scale, where 1 corresponds to "Total disagreement" and 5 corresponds to "Total agreement," students indicated their level of agreement with various statements (see Figure 2). A vast majority of the students, 83%, enjoyed learning about the life of King D. Carlos, expressing levels 4 and 5 of agreement with this statement (green coloured bars in Figure 2). Likewise, the various connections established in the task sparked the curiosity of most students: the connection between the sea and mathematics (70%); the connection between scientific illustration and mathematics (68%); the connection of mathematics to the study of fish (64%); and the connection between the construction of boat models and mathematics (64%).

The majority of the students found it easy to solve part 1 (91%) and part 2 (85%) of the task, while the difficulty level of part 3 divided opinions, with the majority enjoying the challenge posed by that part (47% positive opinions vs 28% negative opinions). Except for this statement, the percentage of negative opinions (orange-coloured bars in Figure 2) was always relatively small. A vast majority of the students, 87%, considered that the task helped them to learn.

Figure 2. Results of the questionnaire about the task "The oceanographer king".



Upon requesting an overall evaluation of the task "The oceanographer king," 47% of the students rated it as Very Good, 40% as Good, 11% as Fair, no students as Poor and only one student, 2%, as Very Poor. Among the comments written by the students in the open-ended question of the questionnaire, the following stand out as representative:

- "The connection between King D. Carlos, the illustrations, and the subject matter we had to cover was quite creative."
- "I enjoyed completing the activity and the combination of mathematics with oceanographic study."

- "Very good, I had fun during the learning process."
- "The activity was great, and I learned a lot."

The analysis of the results described so far aligns with the direct observations made by the teacher.

4. Conclusions

Along the survey, the vast majority of students acknowledged that "the tasks helped them to learn", corroborating the conclusions drawn by the teacher from the analysis of field notes and written productions by the students. Additionally, it became evident that framing mathematics problems in real-world contexts related to the sea had a positive effect on student motivation.

The majority noted that "the connection between the sea and mathematics sparked curiosity" and emphasised enjoying learning about the subject that contextualised each task.

From the analysis of the implementation of the tasks proposed in "Mathematics and the sea" (Frade, 2023, pp. 145-182), the following aspects stand out:

- The established connections enabled students to recognise the relevance of mathematics, understanding that it is part of reality and not just an isolated discipline.
- Students engaged with curiosity and dedication in task completion.
- They identified incorrect resolution strategies and self-corrected.
- Transversal mathematical skills were developed, and other areas of knowledge were deepened, going beyond the learning of specific mathematical content.
- The tasks provided moments of reflection and sharing experiences of active citizenship in support of the ocean.

Finally, the authors express their wish to share and disseminate the tasks proposed in Frade (2023), hoping they can be implemented and/or adapted in other schools throughout the world, for the benefit of students' mathematical learning and the sustainability of the ocean.

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References

Frade, S. (2023). *A Matemática e o mar: conexões para o ensino da matemática no 3.º ciclo do ensino básico* (Master's thesis, University of Aveiro, Portugal). Retrieved from https://ria.ua.pt/handle/10773/39748

Girard, A. (1908). O oceanógrapho e o ichthyologista. In *S. M. el-rei D. Carlos I e a sua obra artística e scientífica* (pp. 67–78), Lisboa: Antonio Palhares. Retrieved from https://archive.org/details/smel reidcarlosie00orti/mode/2up

Ministério da Educação. (2021a). *Aprendizagens essenciais: Matemática: 7.º ano: 3.º ciclo do ensino básico.* Retrieved from https://www.dge.mec.pt/sites/default/files/Curriculo/Aprendizagens_ Essenciais/3_ciclo/ae_mat_7.o_ano.pdf

Ministério da Educação. (2021b). *Aprendizagens essenciais: Matemática: 8.º ano: 3.º ciclo do ensino básico.* Retrieved from http://www.dge.mec.pt/sites/default/files/Curriculo/Aprendizagens_ Essenciais/3_ciclo/ae_mat_8.o_ano.pdf

Ministério da Educação. (2021c). *Aprendizagens essenciais: Matemática: 9.º ano: 3.º ciclo do ensino básico*. Retrieved from http://www.dge.mec.pt/sites/default/files/Curriculo/Aprendizagens_ Essenciais/3_ciclo/ae_mat_9.o_ano.pdf

Morin, E. (2002). *Os sete saberes para a educação do futuro*. Instituto Piaget.

National Council of Teachers of Mathematics. (2008). *Princípios e normas para a matemática escolar* (2nd ed.). Lisboa: Associação de Professores de Matemática.

Yin, R. (1994). *Case study research: design and methods*. Thousand Oaks: Sage Publications.