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CONCEPTUALIZATION OF A DIAGNOSTIC TEACHING DISCOURSE: A CASE STUDY

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Abstract

The purpose of the current empirical research, which departs from a socio-psychological perspective in terms of the concept of diagnostic teaching, is to explore and analyze mathematics teachers' reflections about students' learning of algebra. The theoretical approach proposed by Bernstein and Miller, and the concept of inverse operations in algebra was applied. In this theoretical context, the purpose of developing a diagnostic framework is to include a conceptual triangulation of three domains: (1) conscious inclusion in the teaching environment of a component that addresses students' different levels of prior knowledge and mathematical experience; (2) teacher-student content-oriented conceptual communication; and (3) a gradual sequencing of teaching content. The methodological approach is a mathematical intervention, based on the main principles of the diagnostic framework conceptualization that is rooted in the Swedish PUMP project. The methodological tools are based on qualitative research and data collection from secondary school teachers' group discussions, teachers' analysis of students' diagnostic tests, and written reflections. The participants comprise three teachers and 94 students. The crucial content of the study is a conceptual transition from whole and rational numbers to algebraic equations and problem-solving. The findings of the study illustrate that teachers are aware of the importance of diagnostic teaching and the implementation of diagnostics in the teaching of algebra, practical awareness of students' knowledge of algebra, and what should be taught. The results of the study show teachers' practices and the challenges they face with handling mathematical content and content-oriented communication. The findings clearly indicate that diagnostic teaching that addresses students' mathematical needs and teachers' reflective teaching practice directed at student learning have not been fully developed in practice.

Keywords: *Diagnostic teaching of algebra,* the *PUMP -project, mathematical intervention, content-oriented communication.*

1. Introduction

The importance of diagnostic teaching and its implementation in mathematics classrooms has been described by several researchers (Bell, 1993; Booth, McGinn, Barbieri, & Young, 2017). The key idea behind the conceptualization of teaching using a diagnostics-based approach is to effectively identify the student learning process in a formative way. A prerequisite for learning mathematics is the students' continuous cognitive development and knowledge of mathematical concepts and methods (Davydov, 1990). A diagnostic teaching approach can be helpful for teachers to plan teaching with a focus on students' development of knowledge and help them adjust their misconceptions of mathematical concepts (Galeos, Magno, Galeos, & Apdian, 2024).

2. Background

The current learning discourse of mathematics is complex, and teachers need to be more involved in the mathematical development of their students. In terms of a diagnostic framework, a conceptual structure of teaching content is crucial in order to achieve a dynamic transition in the students' learning process. Researchers claim that it is still the quality of mathematics instruction and the conditions for students' learning of mathematics that need to be further explored (Cohen, Ruzek, & Sandilos, 2018; Friesen & Dreher, 2024). At the same time, students' knowledge development is situated and depends, among other things, on optimal coherence when sequencing the teaching content based on the teachers' explanations and communication (Planas, & Pimm, 2024). There are also current research discussions about the importance of organizing knowledge-oriented content in the teaching of mathematics directed at the development of students' mathematical cognitive functions and mathematical achievements (Brophy, 1986; Stein & Lane, 1996; Thiessen & Blasius, 2008; Chen et al., 2022). Thus, it is important to look back at the history of mathematics education in order to reflect and analyze what has been achieved in the past, pay attention to the contributions of scholars, and identify what is unfinished and should be developed in current research that focuses on teachers' teaching practice. The background to the analysis is also based on a new proposal to reform teacher education in Sweden (SOU 2024:81), as well as the Swedish National Agency for Education preparing courses for teachers' professional development with a focus on students' mathematics development. What do we need to learn in order to develop mathematics education research and teaching practice?

In a study by the innovative Swedish PUMP -project (Kilborn, 1979), particular emphasis was placed on students' difficulties in learning arithmetic and how to individualize the teaching of mathematics using a diagnostic approach, as well as the sequencing of teaching arithmetic content in order to achieve optimal student learning. The roots of the PUMP project were the conceptualization of diagnostic teaching concerning research-based subject-specific content, with particular focus on how this content should be communicated to students. The PUMP project significantly impacted Swedish research into mathematics education, the construction of a successful 1980 mathematics curricula for primary school, and the production of textbooks and constructive guidelines for teaching practice in school using diagnostic matrices (Prytz, 2020). This current research project aims to reinvent the theoretical-methodological perspective that was used in the PUMP project. Against this background, it is important to incorporate diagnostic discourse into teaching practice.

The purpose of the current empirical research, which departs from a socio-psychological perspective related to the concept of diagnostic teaching, is to explore and analyze mathematics teachers' reflections about students' learning of algebra through integration and using a diagnostic teaching discourse. The study is part of a research project about teachers' professional development in a "research circle" (within the framework of the ULF -project).

3. Methodology

The methodological tools are based on qualitative research and data collection obtained from secondary school teachers' group discussions, teachers' analysis of students' diagnostic tests, and their written reflections. The participants comprise three teachers with their 94 students. The crucial content of the study is a conceptual transition from whole and rational numbers to algebraic equations and problem-solving. The teachers received guidance from the researchers to construct and analyze diagnoses, as well as to construct teaching content as a transition from arithmetic to algebra. The crucial arithmetic content is whole and rational numbers, and potencies. The algebraic content is algebraic expressions, rational and linear equations.

4. Theoretical background

The theoretical approach proposed by Bernstein's (1971) and Miller's (1969), concept of inverse operations in algebra (Karlsson & Kilborn, 2024a; 2024b) was applied. In this theoretical context, the meaning of constructing diagnostic framework is to include a conceptual triangulation of three domains: (1) conscious inclusion in the teaching environment of a component that addresses students' different levels of prior knowledge and mathematical experiences; (2) teacher-student content-oriented conceptual communication; and (3) a gradual sequencing of teaching content.

In order to include the teaching environments component as students' different level of mathematical development led to two researchers as Basil Bernstein and George Miller. With his theory of language codes, Bernstein (1971; 1972) studied communication as language codes and found socio-linguistic inequalities based on social strata. The theory is that there are elaborated and restricted codes of communication within the broader category of language code of communication. This explains why students from the lower social strata had to follow the teaching of mathematics with a very restricted code of communication. Another example, from Bernstein's research related to teaching slow learners, was that they became confused when an algorithm required multiple mental(cognitive) tasks before notation of a difference or product. Miller (1956; 1957; 1969) explained this by referring to the capacity of the working memory. In his research, he claims that the capacity of the working memory is 7 ± 2 units of information. For a slow learner, the capacity is just 5 units. When learning algorithms, this will cause serious problems that could be solved using two strategies. The first strategy is to teach in a such way that multiply operations will be collected into a few chunks that can be processed in the working memory. The second strategy is to store some important chunks like subtraction and multiplication tables in an easily accessible way in the long-term memory. Bernstein's and Miller's theoretical perspectives are crucial domains for the conceptualization of the teaching-learning environment, teacher-student communication, and the sequencing of the teaching content in this research project, and previously in the PUMP project. The PUMP diagnostic matrices broke down arithmetic content into units and organized them in two-dimensional patterns called *matrices*. In these matrices it is possible to follow how content develops from the simplest to the most complex, on a step –by –step basis. Bernstein's research was used for the diagnostic teaching of arithmetic, the sequencing of arithmetic tasks and, not least, studying the outcome of teaching by comparing what was taught with what was learned. The same principle is used in the current study with a focus on the teaching of whole and rational numbers by inverse operations (Karlsson & Kilborn 2024b), and how the conceptual transition from arithmetic to algebra can be taught diagnostically (Karlsson & Kilborn, 2024a).

5. Results

An outcome of the present study is teachers' focus on teaching algebra tasks from the current textbook, not on algebraic content concerning students' individual capacity for learning.

The teachers' diagnostic tests regarding students' knowledge of algebra was designed as "summative". When analyzing the diagnostic test results, the teachers focused on correct answers, not on algebraic methods or concepts. In order to support a student who lacks knowledge of algebra, it is important to continuously identify the student's understanding of algebraic methods and concepts. However, this assumes that diagnostic teaching is based on teachers being able to analyze the conditions for students' learning, as well as their algebraic challenges and needs.

The results clearly show that teachers lack experience of working with the diagnostic approach in their teaching and are not trained to carry out diagnostic tasks or activities. In addition, teachers were unable find any diagnostic algebraic tasks in the current textbook that focused on didactic structure of conceptual algebraic content by given algebraic tasks. Against this background, it is interesting to study the proposal for diagnostic tasks in the current textbook. The section about fractions begins with many formulas about how to operate with fractions. However, many of the students had already encountered these formulas before and failed to use them. A smarter approach would have been to define the concept of fractions, because many students think of fractions as division, which causes a lot of misunderstanding. With correct definitions like $\frac{a}{b} = a \cdot \frac{1}{b}$, $b \neq 0$, it is easy to understand all formulas by using the same laws of arithmetic that the students learned in grades 1–3.

Another important outcome of the current research is teachers' reflections on diagnostic teaching: the more exercise with algebraic tasks, the more learning. Thus, according to the interpretation of the teachers' reflections, if a student, for example, is unable to solve a certain kind of equation due to a misconception an exercise by training of such tasks will rather strengthen that misconception. This means that there is a need for a clear teaching plan with a priority of content and sequencing of algebra content, students' clear instruction, and systematic diagnostic guidelines.

6. Conclusions

The findings illustrate that teachers are aware of the importance of diagnostic teaching and the implementation of diagnostics in the teaching of algebra, practical awareness of students' knowledge of algebra, and what students should be learnt. At the same time, the results of the study show that teachers' diagnostic practices and experience of diagnostic teaching, as well as algebraic content and content-oriented communication is limited. The findings clearly indicate that diagnostic teaching that addresses students' learning of algebra, as well as teachers' reflective diagnostic teaching practice directed at student learning, have not been fully developed in practice. We should consider studying this problem on a deeper level. When a students' struggles because they lack knowledge, they will probably struggle forever if they do not receive qualitative systematic diagnostic support. There is, however, another explanation. For social reasons (Bernstein, 1971; 1972), limited working memory (Miller, 1956; 1967) or lack of aptitude for learning mathematics, some students will never be able to solve tasks of a certain level of complexity, maybe due to time constraints. This often involves 50% of students and there ought to be differentiating curricula, with different learning goals that allow students to study according to their ability. With framework of the PUMP project, Kilborn (1979), created a tool for constructing individual learning goals for such students in the form of teaching matrices. The experiences from the PUMP project, specifically these teaching matrices, can be useful for designing a diagnostic discourse for teaching and provide the necessary support for teachers.

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