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NEW APPROACH TO MATHEMATICS TEACHING BASED ON THE CULTURAL ONTOLOGY

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Abstract

Curricular tradition often distills teaching to convey only the "correct" knowledge for a specific domain at a particular level, time, and place, as seen in commonly used textbooks. The development of knowledge, if mentioned at all, typically outlines steps leading to specific knowledge while avoiding debate with alternatives. For instance, in teaching geometry, figures like Euclid, Pythagoras, and Thales are highlighted, whereas Lobachevsky, Bolyai, and Riemann are often omitted, despite their significant contributions. By excluding these figures, we deprive students of a deeper understanding of the discipline they are learning. A linear, distilled curriculum presents correct knowledge as an undisputed path to learning. However, this approach is flawed regarding meaningful learning. The disciplinary policy frequently contrasts the tradition of scientific progress (e.g., Matthews, 2015) and contradicts essential learning conditions (e.g., Marton, 2015). In this study, eight third-year undergraduates—future junior high mathematics teachers—were asked to define mathematical contexts shared by mathematics and physics (e.g., derivatives), contexts relevant only to one discipline (e.g., irrationality in mathematics), and content lacking consensus across different mathematical theories (e.g., parallel lines). They then learned about various historical definitions of the concept of angles and how these definitions influence current mathematics curricula and textbooks at all school levels. The DC-Structure (Tseitlin & Galili, 2005) defines the term "ontology" in both philosophical and computational senses (Zhitomirsky-Geffet, 2016) and provides examples of cultural ontologies in Euclidean and Non-Euclidean Geometries (CCK - Content Knowledge). Undergraduates were asked to add the basic concepts, axioms, and results of Non-Euclidean Geometry to the existing structure, distinguishing it from Euclidean Geometry. After this learning experience, they redefined the mathematical contexts they initially defined. The results demonstrated varied complexity. Some students found it challenging to recognize differences within the mathematical content, while others acknowledged the importance of such recognition for future mathematics teachers. They appreciated the experience for fostering meaningful learning of disciplinary knowledge and its impact on teaching practices. Some undergraduates reported that the experience helped them recognize connections between their courses and shifted their views and perceptions of the knowledge they acquired. The research was conducted based on constructive-qualitative methodology (Shkedi, 2011).

Keywords: *Discipline culture, ontology, meaningful learning, geometry.*

1. Introduction

Conceptual understanding goes beyond rote memorization or the mechanical application of procedures. It entails grasping core ideas, enabling the transfer of knowledge to analyze new scenarios and adapt it to different contexts. Rather than accumulating isolated facts or methods, it requires organizing knowledge elements into a coherent structure. This structure should have a clear hierarchy, define their relationships and affiliations, and establish their validity and reliability, fostering deeper insight and practical application.

The curricula of school geometry often fall short of meeting modern educational demands. Confronted with an exponential growth of knowledge, educators frequently resort to requesting additional teaching hours to accommodate more content. However, while this approach appears reasonable, it is insufficient to address the broader challenge of the information explosion. We propose adopting a new educational paradigm that transforms a discipline into a discipline-culture (DC) (Tseitlin & Galili, 2005).

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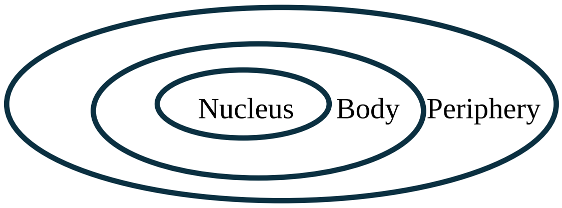
According to Galili and Tseitlin physical theory is an inclusive construct incorporating a large number of knowledge elements within certain paradigm with certain domain (e.g. classical mechanics, electromagnetism, optics, etc.) The authors claimed (ibid.) that:

> Any fundamental physical theory arranges all statements in a centralized structure, a sort of quasi-culture, with its own values, language, conceptions, norms, etc. This structure, like a cell, has nucleus, body, and periphery.

Figure 1. Symbolic representation of areas of knowledge elements within Discipline-Culture structure of a fundamental theory (Tseitlin & Galili, 2005).



In this structure (Figure 1), the nucleus of the theory includes fundamental principles, ontological and epistemological, and the paradigmatic model of the theory. Body elements are all those knowledge elements coherent with the nucleus (theoretical and empirical) and explained by it. The periphery includes the elements of knowledge at odds with the nucleus (alternative accounts and concepts from other disciplines, old and new, unsolved problems, unexplained phenomena). It is the periphery that introduces a dialogue between different theories making disciplinary knowledge cultural. It also makes explicit the limits of validity of the particular theory. Thus, while learning the Newtonian account of motion in mechanics, the students become aware of the alternative concepts of motion in earlier accounts by Aristotle and Philoponus as well as the pertinent ideas from relativistic and quantum theories. This way the considered concepts learned obtain a space of conceptual variation essential for conceptual understanding (Marton, 2015). Our understanding consolidates in comparison with alternatives, discerning the essence of the concept from the differences. A key feature of this paradigm is its structured hierarchy, which organizes knowledge and fosters conceptual discourse on the essential content of each discipline. Emphasizing the creation of Cultural Content Knowledge (CCK) (Galili, 2012) promotes conceptual understanding. In teaching geometry, figures like Euclid, Pythagoras, and Thales are highlighted, whereas Lobachevsky, Bolyai, and Riemann should be presented with their significant contributions to geometry. CCK approach uncovers and clarifies the underlying principles and meanings, guiding learners toward deeper comprehension and a more integrated perspective of their studies.

This research focuses on developing cultural ontologies (NCO) for geometry, grounded in the theoretical framework of the "Discipline-Culture" paradigm proposed by Galili and Tseitlin (2005). The term "cultural" emphasizes the plurality of perspectives, highlighting the conceptual variation inherent in disciplinary knowledge. This pilot study aims to explore the impact of the Cultural Content Knowledge (CCK) approach on teaching geometry to pre-service mathematics teachers.

2. Research

2.1. Research design and methods

The research applies the new DC paradigm to the disciplines of the school curriculum in geometry. The Euclidean and non-Euclidean geometry could be introduced the triadic discipline-culture structuring of the curricula varying in the contents and nature of primarily the nucleus constitutional elements, yet sharing the principle of facilitating conceptual understanding through the comparative analysis of knowledge elements with peripheral alternatives.

The research questions address applicability of the DC paradigm, the quality of conceptual understanding of the subject matter, and pre-service mathematics teachers views on the nature of the knowledge they learn.

In terms of specific points, we may mention the following:

1) The development of learning-teaching materials for their conceptual understanding based on the developed DC paradigm and cultural ontology. In the field of geometry.
2) The development of a course for pre-service teachers introducing them to the new DC paradigm, the building of cultural ontology, and forms of teaching of the pertinent materials.
3) Experimental training of teachers for the new educational paradigm in colleges of education for per-service high school mathematics teachers.

4) Measures for assessment of quality of the developed learning materials and cultural ontology, and its effectiveness and contribution to conceptual understanding of the pre-service high school mathematics teachers.

2.2. The research questions

The research questions will address the applicability of the DC paradigm in the geometry, the quality of conceptual understanding of the subject matter, and pre-service high school mathematics teachers' views on the nature of the knowledge they learn, changed due to the creation of cultural content knowledge.

1) Whether and to what extent does the DC paradigm promote conceptual understanding in geometry?
2) What is the quality of conceptual understanding of the subject matters promoted by the DC paradigm?
3) How have the pre-service teachers' views on the nature of the knowledge they learn changed due to the creation of cultural content knowledge?

2.3. Methodology

2.3.1. Empirical data collection and analysis. The qualitative research which is based on criteria-focused methodological patterns, placing increased emphasis on the analytical research skills and expertise at the expense of intuition was conducted. The significance of this fact is that the researchers work and rely on a system of external criteria (theory, principles, and categories) throughout the whole study. This pattern is close to basic positivistic assumptions in quantitative research, but does not repeat them (Shkedi, 2011, p. 118). The criteria-based methodology reflects intuitive and impression-based resources. In the realm of qualitative research, researchers use analytical skills but seek to express phenomena in verbal terms often turning to philosophical aspects. The methodology presented here belongs to qualitative research and includes all the methodological components that characterize such research (Shkedi, 2011).

2.3.2. Cultural ontology of geometry was construction. This research focuses on developing a methodology and guiding principles for domain experts to create a network of interlinked cultural ontologies (NCO) for geometry based on the theoretical framework of "Discipline-Culture" proposed by Galili and Tseitlin (2005).

The methodology consists of several stages. Initially, we constructed a single-theory or single-viewpoint ontology. The ontology is drawn from subject knowledge and notable references, such as textbooks and encyclopedias. The ontology comprises a nucleus (the fundamental elements of a discipline's theory) and the body of knowledge built upon the nucleus.

The existing generic domain ontologies were extended to achieve this. For example, in Mathematics, the upper-level ontology OMDoc (Lange, 2013) includes concepts like Math Knowledge Item, Theory, Axiom, Definition, Proof, and Statement. The semantic relationships between ontology classes include hierarchical connections (e.g., subclass-superclass), inclusion (e.g., part-of, substance-of), and entailment (e.g., cause-of, use-of). Finally, all ontology elements were annotated with newly introduced properties, "nucleus" and "body," based on their position within the theoretical knowledge structure.

3. Results and discussion

3.1. First stage questionnaire

Eight third-year undergraduates—future junior high mathematics teachers—were asked to define mathematical contexts shared by mathematics and physics (e.g., derivatives), contexts relevant only to one discipline (e.g., irrationality in mathematics), and content lacking consensus across different mathematical theories (e.g., parallel lines). The answers they provide show that there was no awareness of the plurality of mathematical theories, variation of definitions and diachronic discourse.

3.2. Second stage learning

1) Eight third-year undergraduates learned about various historical definitions of the concept of angles (Barabash, 2016) and how these definitions influence current mathematics curricula and textbooks at all school levels.
2) They were introduced to the DC paradigm and CCK examples (Vinitsky & Galili, 2014a, 2014b, e.g. Levrini et al., 2014).

3) They were introduced to the NCO and the example provided by the lecturer (Zhitomirsky-Geffet, M., & Eden, 2014). The lecturer had to write a reflection on the process she was going through with students. The students were surprised since they were used for polyphonic representation of the disciplinary content.

3.3. Third stage implementation

Eight third-year undergraduates were asked to imply the knowledge they were exposed to by doing a summary assignment in the course (Figure 2).

Figure 2. Summary Assignment in the Course: Teaching Geometry.

Summary Assignment in the Course: Teaching Geometry

The purpose of this assignment is to actively involve you in constructing a multi-viewpoint ontology in geometry.
You are provided with an Excel file in which I have begun creating a multi-viewpoint ontology encompassing Euclidean Geometry, Synthetic Geometry (as covered in the axioms section of the course), and Lobachevskian Geometry. Your task is to expand upon this work by starting the construction of an ontology for Spherical Geometry.

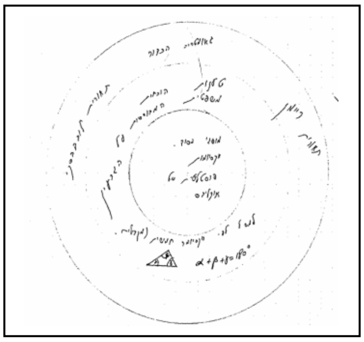
1. *List the fundamental concepts of the theory, if applicable.*
2. *Document the axioms of the theory, if they exist.*
3. *Write three propositions that hold true in both Synthetic Geometry and Spherical Geometry, if applicable.*
4. *Write three propositions that hold true in Synthetic Geometry but not in Spherical Geometry, if applicable.*
5. *Write three propositions that do not hold in Synthetic Geometry but do hold in Spherical Geometry, if applicable.*
6. *Indicate any "derives-from" relationships between the propositions and axioms, if applicable.*
7. *Specify relationships such as "inclusion," "overlap," and so on, if applicable.*

Good Luck!

The students were divided into three work groups for this assignment, with group sizes of 3, 3, and 2. All the groups successfully completed the task, though they noted that it posed significant challenges from the mathematical perspective. To overcome these difficulties, they frequently consulted a variety of academic sources.

The students' performance in this assignment highlighted how Discipline-Culture (DC)-based materials foster dialogue between ideas, encouraging learners to engage in the discourse of geometric knowledge. Below is an explanation from one of the students about how they utilized the DC structure to organize their understanding:

Figure 3. Participant 5, DC- structure of geometry.



"*We can organize the knowledge for the students in the way that when they learn something new (a definition or theorem) we check where it is placed (in DC) and understand how it is connected to the principles…. If we change a definition of a principle we get another geometry – Non Euclidean…" (participant 1)*

"The nucleus-basic consepts of the Euclidean geometry. The body – sum of triange engles 180 degerres. The periphery - spherical geometry, "Riemannian geometry"..." (participant 5, Figure 3)

Correct answers and successful performance on the assignment reflect the pre-service teachers' conceptual understanding. The cultural ontology, grounded in multiple theories and approaches, provided them with exposure to diverse definitions and connections across various geometries. These differences, along with the hierarchical structure of the discipline, were evident through the ontology database. Based on the pre-service teachers' responses, their views and perceptions of the nature of the knowledge they were learning underwent a noticeable shift.

4. Conclusion

This pilot research encompassed three stages, during which we observed the transformations undergone by pre-service high school mathematics teachers. Although the mathematical component of this course was challenging for them, the students (who were pre-service teachers) gained significant benefits from the cultural approach in several aspects:

1) They learned in an open-minded and cultural learning environment which will place the knowledge in context and see the holistic picture of knowledge and possible practical applications of the acquired theoretical knowledge in different related disciplines.
2) By mining the ontology, they explored intra- and inter-disciplinary relationships and were able to comprehend the foundations and evolution of theories and learn about cross-theory relationships.
3) They were exposed to a diversity of theories and viewpoints and learnt the grounds and reasons for similarities and differences between them.

This study is one among several that will soon follow, featuring a larger sample size, exploring various mathematical subjects, and addressing different research objectives.

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