

## INCORPORATING GEOGEBRA DISCOVERY INTO TEACHER TRAINING FOR THE PLANE GEOMETRY STUDY

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### Abstract

The teaching of Plane Geometry is one of the pillars of Mathematics Education, and its understanding is essential for the development of students' logical reasoning. However, the way this content is taught can be significantly improved with the use of digital technologies, such as GeoGebra Discovery, an experimental version of the GeoGebra software that incorporates automated reasoning tools, known as Automated Reasoning Tools (ART). These commands allow teachers and students to explore geometric properties in greater depth, visualizing relationships and verifying theorems directly in the software, allowing them to verify conjectures and perform automatic proofs in geometric constructions, making the teaching and learning process more efficient, more interactive and dynamic. This article explores the use of GeoGebra Discovery software in continuing education for Mathematics teachers and investigated how GeoGebra Discovery's automated reasoning tools (ART) can contribute to teaching properties of Plane Geometry. The theoretical basis of the study is based on the Technological Pedagogical Content Knowledge (TPACK) theory, proposed by Mishra and Koehler (2006). This model recognizes that the effective use of technology in teaching requires a coherent integration between content, pedagogical and technological knowledge, and this article discusses the software's potential for mobilizing this knowledge. Considering the Design Research methodology, which allows the creation, execution and analysis of educational interventions in real contexts, the continuing education was organized in synchronous meetings, held via the Microsoft Teams platform, in which participants carried out practical activities with the software, developing geometric constructions and using automated reasoning commands to verify properties and theorems, mobilizing technological, pedagogical and content knowledge. Among the challenges faced, the initial difficulties in using GeoGebra Discovery commands and the need for more practice time to master the software stand out. However, teachers reported that the use of ART facilitated the understanding of geometric concepts and brought new possibilities to their teaching practices. The use of GeoGebra Discovery in the teaching of Plane Geometry has several didactic implications because it allows students to explore geometric properties interactively, visualizing relationships that would be difficult to perceive in static constructions. The results of the research demonstrate that the use of GeoGebra Discovery and similar digital tools should be encouraged in educational contexts, as a way to enrich the teaching of mathematics and promote the development of essential skills for the 21st century.

**Keywords:** *Plane geometry, GeoGebra discovery, teacher training, mathematics education.*

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### 1. Introduction

Technological advances have impacted Mathematics Education, providing new methodological possibilities for teaching and learning. In this context, Dynamic Geometry Systems (DGS) stand out as innovative tools that allow an exploratory interactive approach to the study of Geometry. Unlike traditional methodologies, which use static representations, DGS enable the dynamic manipulation of geometric figures. This dynamism favors not only a greater understanding of geometric concepts but also stimulates critical thinking and student autonomy in the learning process (Lima & Rocha, 2022), relevant skills to be developed in the 21st century.

However, initial teacher training courses have not prepared teachers adequately, nor have they satisfactorily used the potential of digital technologies (Colling & Richit, 2019). Discussions and training take place in generic ways and are poorly contextualized with real situations (Santos & Vasconcelos, 2019).

In this work, GeoGebra Discovery was used as DGS, an experimental version of the GeoGebra software that incorporates a set of automated reasoning tools, the Automated Reasoning Tools (ART), of which the following were used in this study: *Relation*, *LocusEquation* and *Discover*. These commands

allow the user to automatically conjecture, discover, prove and verify statements about different elements of a given geometric construction, for example, congruence relations, inequalities, perpendicularity, parallelism of segments and concyclic points, as well as generate the implicit equation of a geometric locus associated with a property to be validated (Kovács, Recio, & Véllez, 2022).

The objective of this work is to present possibilities of how to incorporate the automated reasoning tools (ART) of GeoGebra Discovery in the training of mathematics teachers, since they can contribute to the teaching of properties of Plane Geometry, in addition to presenting the possible difficulties faced during this process.

In this context, the theoretical contribution, the TPACK model (Mishra & Koehler, 2006), provided conditions to explore the technological and pedagogical knowledge of the content that is mobilized by mathematics teachers during the manipulation of the GeoGebra Discovery software when developing the proposed activities. It was found that the dialogue developed during the proposed activities, the answers provided by the teachers to the questions asked by the trainers and the doubts shared by them in the meetings, in addition to the tasks produced, made it possible to conduct the analyses.

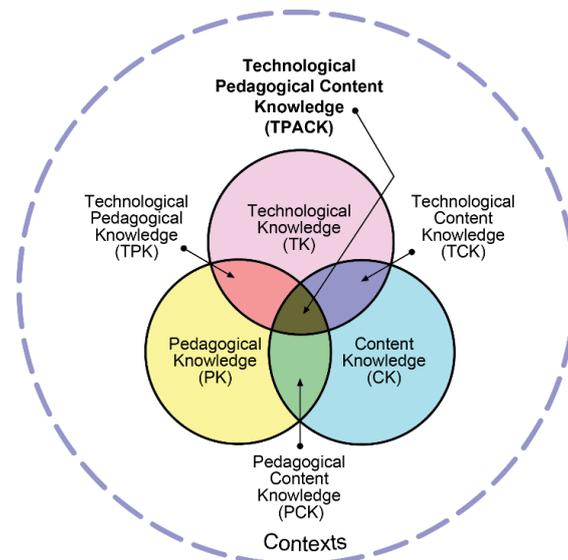
## 2. Theoretical framework

The TPACK (Technological Pedagogical Content Knowledge) theory, developed by Mishra and Koehler (2006), aims to analyze the integration between technological and pedagogical knowledge linked to the specific content to be taught. This approach enables understanding and description of the different types of knowledge, which is essential for a teacher to develop an effective pedagogical practice in technology-mediated learning contexts.

The authors emphasize in their work that technologies are constantly changing, an inherent characteristic of technological knowledge, which makes it essential for teachers to continually update themselves for their professional development (Koehler & Mishra, 2009). This model was conceived as an expansion of the concept of pedagogical content knowledge (PCK), introduced by Shulman (1986).

Figure 1 presents the components of the TPACK model and their intersections, all of which are part of a broader context that encompasses variables that are external to the control or specific knowledge required of teachers to teach. Furthermore, Koehler and Mishra (2009, p. 66) add that the set of knowledge that teachers need to develop encompasses “knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face”, as well as the technologies that can be used to strengthen students’ old knowledge and enable the creation of new epistemologies.

Figure 1. The TPACK framework and its knowledge components.



## 3. Methodology

This qualitative study used the Design Research methodology, characterized by Collins, Joseph and Bielaczyc (2004) as an approach aimed at conducting research and improving educational projects, based on empirical evidence. In addition, this methodological support allowed adjustments to be implemented throughout the process.

This methodology permeated the creation, execution and conduction of the training course, as it makes it possible to observe points for improvement and implement improvements during the investigative process – redesign – based on feedback from the five Basic Education and Higher Education teachers in Brazil, and in the trainers' meetings.

Nine synchronous meetings were held, each lasting an hour and a half, via the Microsoft Teams platform, and the data was produced by recording the interactions that occurred during the meetings, which included the sharing of screens and dialogue between participants, in addition to viewing the development of the proposed activities.

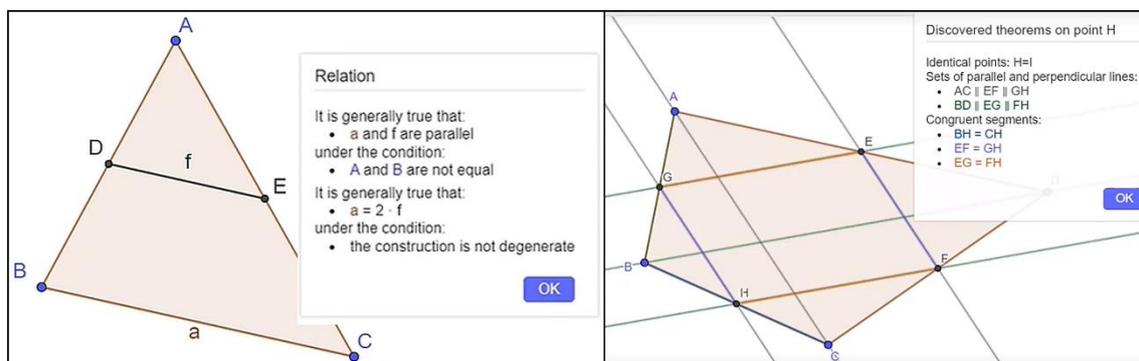
#### 4. Discussion

To explore one of the proposed activities, which addressed the midpoint of a triangle and its reciprocal, the *Relation* and *LocusEquation* commands were used. In this context, the theorem can be stated as follows: the segment that joins the midpoints of two sides of a triangle is parallel to the corresponding base and has half its length. In addition, in the development of another activity, a question was formulated to the teachers, starting from the construction of any quadrilateral, in order to investigate the properties and the figure formed from the midpoints of its sides.

The first activity allows teachers to work with the question of how to motivate students in the classroom in the ability to formulate conjectures and, through the *Relation* tool, verify the validity of the relations (Abar & Souza, 2024). Regarding the second proposal, it is worth noting that, at first, the teachers had not understood or visualized that the figure formed was a parallelogram, and not just any quadrilateral.

Figure 2 shows on the right the result of the search for some relation between segments  $a$  and  $f$ , using the *Relation(a,f)* command. On the left, the results provided after processing the GeoGebra Discovery ART are shown, using the *Discover(H)* command. It should be noted that, to use this last resource, it is necessary to choose a point of construction to apply it, and the possible properties found will be related to this point.

Figure 2. Results of the ART responses in the triangle and quadrilateral activities.



Souza (2024) highlights the possibilities of using the *Discover* command to work on discovering geometric properties in flat figures, which involve congruence, parallelism and perpendicularity of segments and lines. This resource has the potential to develop geometric visualization skills, which allows exploring the identification of relevant geometric information in the figure, through the interpretation of the data provided by the software.

Arcavi (2003) considers that a context, in a situation involved, can enable the articulation of the formation of mental and non-mental images in the development of geometric visualization skills. In this sense, observing the TPACK model (Mishra & Koehler, 2006), evidence of the mobilization of Technological Content Knowledge (TCK) by mathematics teachers stands out, when conjecturing and thinking of possible paths, to define the location of the point that satisfied the property of the average base of the triangle or when they were articulating the interpretation of the information in the *Response Window* of the *Discover* command, with the construction and their own knowledge of the content of Plane Geometry (Souza, 2024).

Design Research Methodology (Collins et al., 2004) made it possible to carry out an analysis of the challenges faced during the training process, in order to seek changes and improvements that would overcome the difficulties experienced. The initial difficulties in using the GeoGebra Discovery commands stand out, since some teachers did not clearly understand how to apply or in which situations to use the *Discover* command. In addition, the need for more practice time to master the software.

## 5. Conclusions

In view of the discussions presented, it is concluded that the use of GeoGebra Discovery ART, in particular, the *Relation*, *LocusEquation* and *Discover* commands, proved to be an effective strategy for the exploration of geometric properties, contributing to the development of teachers' geometric visualization skills, during the continuing education process, by allowing them to conjecture, test hypotheses and articulate different concepts of Plane Geometry.

However, for the integration of these digital tools to be effective in the teaching process, it is necessary to consider the challenges and limitations involved. In this sense, the TPACK model considers the so-called contextual factors, for example, institutional bureaucracy and access to technological support, whether computers or internet connectivity. It is necessary that these elements be considered by teachers when developing their plans, since, in addition to specific training for mathematics teachers, it is important that they are able to prepare activity proposals that articulate the programmatic contents with the technological resources and the need to observe the contextual factors mentioned.

The data showed that the teacher training process, involving digital technologies, needs to be active and for a long time, that is, asking teachers to reflect on their practice, to produce weekly tasks that refer to their experiences in the classroom (Souza, 2024).

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