THE EFFECT OF LENGTH MEASUREMENTS DIGITAL GAMIFICATION ON VAN HIELE LEVELS OF GEOMETRIC THINKING

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Abstract

Geometry is one of the basic branches of mathematics that has developed over the years according to the need for objects measurements around us. Van Hiele theory is one of the essential theories in teaching geometry which may help to design instructional materials to improve students' mathematics performance. This model describes five levels of geometric thinking, such that it is impossible to bypass or skip a level and move to the next while learning a geometrical content. It comprises five levels: visualization, analysis, informal order/abstraction, deduction, and rigor. The aim of this study is to examine the effect of digital gamification that deal with the length concept on the geometric levels of thinking according to Van Hiele, among 5th graders. This study is an experimental one and was edited by basically quantitative method. A total of 55 fifth graders participated in the current study. Data was collected by two questionnaires; one questionnaire was before an intervention program (15 lessons) based on digital gamification in measuring length and the second questionnaire was after the intervention. Kuder-Richardson 20 (KR-20) was employed to estimate internal consistency of the questionnaires, $r_{KR20} = 0.566$. The main finding of this study was that digital gamification contributes significantly to the development of the Van Hiele thinking levels which deal with the length concept. There was significant progress from low-thinking level before the intervention to a higher level after the intervention; the percentage of students who reached the third Van Hiele level improved, 31% to 55% pre and post the intervention program respectively. Moreover, the visualization thinking level was affected significantly by the intervention in all skills except the logical one; in the analysis thinking level the intervention affected the applied skill prominently. And in ordering level the effect of the intervention on the visual skills was negligible. Based on this study it is recommended to integrate digital gamification in teaching length measurement among students, particularly fifth grade students.

Keywords: Geometry, Van Hiele theory, length measurement, digital gamification.

1. Introduction

Geometry is a basic branch of mathematics, it is based on fundamental concepts, axioms, and properties. To achieve a deep understanding of geometry, the student needs to develop thinking skills related to the subject of geometry; spatial perception is the basis for learning geometry and contributes to the development of geometric thinking (Van Hiele, 1999). Many scholars investigated students' understanding and geometrical thinking, i.e., Astuti & Suryadi (2018) referred to Van Hiele's theory to understand students' geometric thinking levels. This theory has three aspects: the existence of levels, the properties of the levels, and the progress from one level to the next level. According to Van Hiele theory, there are five levels of thinking or understanding in geometry: Level 1 (recognition/visualization)- at this level students start analyzing and naming properties of geometric figures. They do not see relationships between properties. Although they can measure, fold and cut paper, use geometric software etc., they do not see a need for proof of facts discovered empirically. Level 2 (analysis): At this level students perceive relationships between properties and figures, but they cannot explain the relationship between these traits and cannot understand the definition. Level 3 (order/abstraction): At this level students can see the relationship of properties on one or many geometry objects, students can create abstract definitions, discover the properties of various objects using informal deduction, and classify them hierarchically. Level 4 (deduction): At this level students can give deductive geometric proofs. They are able to differentiate between necessary and sufficient conditions. They identify which properties are implied by others. They understand the role of definitions, theorems, axioms and proofs. And level 5 (rigor): At this level students

understand the way mathematical systems are established, can work in different geometric or axiomatic systems, and can compare systems based on different axioms and can examine geometry without presenting a concrete model (Clements, 2003; Moyer, 2003; Prapowo & Eri, 2011; Van de Walle, 2011).

Hoffer (1981) outlined five essential skills for mastering geometry. The first is 'visual skill', which involves the ability to recognize various geometric figures and spatial relationships, identify centers and symmetries, understand how parts relate to the whole, classify shapes, and mentally visualize geometric models. The second is 'verbal skill', which includes the ability to name and describe geometric objects accurately, interpret and construct definitions, express relationships between figures, generalize concepts, and comprehend the logical structure of verbal problems. The third is 'drawing skill', refers to the capacity to sketch geometric figures, label significant points, translate verbal descriptions into diagrams, create figures based on properties, enhance images with relevant elements, recognize the limitations of sketches, and construct geometric models. The fourth is 'logical skill', which entails comparing and classifying geometric objects based on their properties, applying essential definitions, drawing logical conclusions, developing proofs, and understanding the use and limitations of deductive reasoning. Finally, 'applied skill' involves recognizing geometric concepts in real-world contexts, building models from physical objects, applying geometric properties to these models, representing natural phenomena with geometric reasoning, and solving problems using these representations.

In the field of geometry education, early introduction at the elementary level and the implementation of active pedagogical strategies are strongly recommended (Salifu et al., 2018). These recommendations underscore the need for educational researchers to explore innovative teaching methods that can transform traditionally passive and disengaging mathematics classrooms into dynamic, interactive learning environments. One promising alternative to conventional instruction is the adoption of active learning approaches, such as educational games, which have been shown to enhance student participation and conceptual understanding (Nand et al., 2019; Ting et al., 2019). Within this context, 'gamification' - defined as the integration of game elements such as points, rewards, challenges, tasks, and immediate feedback into the learning process (Lämsä et al., 2018)- emerges as a particularly effective strategy. Empirical studies have demonstrated that gamification can yield positive outcomes in mathematics education broadly (Baldeón et al., 2015; Stoyanova et al., 2017), and it has shown specific benefits in the teaching and learning of geometry, particularly in terms of increasing learner engagement and academic achievement (Aridi & Saad, 2020; Kamalodeen et al., 2021).

Length measurement is a fundamental component of the mathematics curriculum. However, Sarama et al. (2022) highlights that students often exhibit significant difficulties in developing a sound understanding of measurement concepts. Common challenges include the inability to determine the length of an object using a ruler without numerical labels, failure to align the object with the zero point of the measuring tool, and a tendency to count hash marks rather than the intervals between them, which leads to an error that frequently results in inaccurate measurements. Additionally, students often struggle to place physical units of length end-to-end along an object without leaving gaps or overlaps, further indicating a lack of conceptual grasp of continuous measurement. Following these claims, it is necessary to investigate students' usage of digital gamification in length measurements.

2. Objectives

The purpose of this study is to examine the effect of digital gamification that deals with the length concept on the geometric levels of thinking according to Van Hiele, among 5th graders.

Research Questions
1. To what extent did 5th graders control the Van Hiele levels and the five Hoffer's skills before digital gamification usage?
2. To what extent did 5th graders control the Van Hiele levels and the five Hoffer's skills after digital gamification usage?
3. How does digital gamification impact the progression of 5th-grade students through the Van Hiele levels of thinking in relation to the concept of length?

3. Methodology

This study was designed by quantitative method.

3.1. Participants

A total of 55 fifth graders (28 males and 27 females) participated in the current study from the north zone of our country. The range of the participants' mathematics average was (60% - 75%). The participants had similar socioeconomic status.

3.2. Instruments

In this study the following instruments were used:

a. Two (pre, and post) mathematical achievement questionnaires related to the "length measurement" content. Both questionnaires included 15 length measurement problems due to the three first van-Heile thinking levels: The first five problems (1-5) test the first level, the recognition level, problems from (6 – 10) test the second level, the analysis level, and problems (11 – 15) test the third level, the ordering level. Kuder-Richardson 20 (KR-20) was employed to estimate internal consistency of the questionnaires, $r_{KR20} = 0.566$

b. An intervention program (15 lessons) based on digital gamification in measuring length. Table 1 details the units and some online digital game length measurement examples.

Table 1. Intervention units' content.

Unit*	aim	Games' sites- example
1	**Measuring length at the recognition level**	• https://www.ixl.com/math/grade-4/measure-using-an-inch-ruler • http://www.softschools.com/measurement/games/ruler/measuring_length_in_cm/
2	**Measuring length at the analysis level**	• http://www.mathplayground.com/geoboard.html • http://phet.colorado.edu/sims/estimation/estimation_en.html • http://www.iboard.co.uk/activity/Measuring-Lines-Make-1m-270 • https://www.ixl.com/math/grade-5/perimete
3	**Measuring length at the order level**	• https://www.ixl.com/math/grade-6/find-side-lengths-of-similar-figures • http://pbskids.org/cyberchase/math-games/sleuths-on-the-loose • https://www.ixl.com/math/grade-3/relationship-between-area-and-perimeter

(*) Each unit includes 5 lessons.

4. Results

Table 2 presents the percentages of students according to Van Hiele levels and Hoffer skills pre and post digital gamification.

Table 2. Percentage of students' right solutions pre and post gamification according to van Hiele levels and Hoffer skills.

van Hiele level	van Hiele Level Control* (%)		Skill	Right Solutions (%)	
	Pre gamification	Post gamification		Pre gamification	Post gamification
Recognition	58	83	Visual	51	96
			Verbal	60	87
			Drawing	38	92.7
			Logical	72.7	63.6
			Applied	67	76
Analysis	55	71	Visual	60	51
			Verbal	69	80
			Drawing	32.7	66.5
			Logical	36	76
			Applied	78	81.8
Order	34	55	Visual	60	71
			Verbal	34.5	41.8
			Drawing	31	56
			Logical	23.6	51.6
			Applied	21.8	55.5

(*) Control means when a student passing from a van Hiele level of geometrical thinking to the next level.

4.1. Research question no. 1

The first research question was: " To what extent did 5th graders control the Van Hiele levels and the five Hoffer's skills before digital gamification usage?"

Table 2 (columns 3 and 5) shows that pre digital gamification van Hiele levels, 58%, 55% and 34% according to recognition, analysis and order respectively. Moreover, the table presents the ranges of each Hoffer skills level: 38%-67%, 36%-78%, and 21.8%-60% (according to the van Hiele first three levels respectively).

4.2. Research question no. 2

The second research question was: " To what extent did 5th graders control the Van Hiele levels and the five Hoffer's skills after digital gamification usage?"

Table 2 (columns 4 and 6) shows that post digital gamification van Hiele levels, 83%, 71% and 55% according to recognition, analysis and order respectively. Moreover, the table presents the ranges of each Hoffer skills level: 63.6%-96%, 51%-81.8%, and 41.8%-71% (according to the van Hiele first three levels respectively).

4.3. Research question no. 3

The third research question was: "How does digital gamification impact the progression of 5th-grade students through the Van Hiele levels of thinking in relation to the concept of length?"

Table 2 presents clear differences in van Hiele levels and Hoffer skills in favor of the post digital gamification usage. Recognition level jumped from 58% to 83%, analysis level jumped from 55% to 71% and the order level jumped from 34% to 55%. In addition, each skill increased significantly for each van Hiele level.

5. Discussion

The findings of this study indicated that digital gamification contributes significantly to the development of the Van Hiele thinking levels which deal with the length measurement concept. There was significant progress from low-thinking level before the intervention to a higher level after the intervention. Moreover, the visualization thinking level was affected significantly by the intervention in all skills except the logical one; in the analysis thinking level the intervention affected the applied skill prominently. And in ordering level the effect of the intervention on the visual skills was negligible.

The current study investigated students' digital gamification usage, which is an innovative teaching method, an interactive learning environment. Thus, our findings may be explained by Salifu et al. (2018) who suggested that games are an active learning pedagogy and transform the dull learning environments in mathematics classrooms into smart ones.

6. Conclusions

This study recommends integrating digital gamification into teaching length measurement among students.

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