

SPATIAL REASONING WITH A HUMAN-SCALED GEOMETRIC PUZZLE

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Abstract

As our world interactions become more digital, youth have fewer opportunities to engage in physical activities that provide contexts for mathematics. Most school math is done on paper or computer screen—2D environments with little opportunity for embodied learning. This paper describes how activities with human-scale geometric objects offer youth opportunities to strengthen, challenge, and contextualize what they have learned in school. Using a human-sized 3D puzzle made of cardboard boxes, youth engaged in embodied learning experiences that involved spatial perception, mental rotation, spatial visualization, and joy. The activity, conducted in an afterschool program, fostered collaboration and deeper engagement through physical manipulation of the puzzle pieces. This case study shows the promise of large-scale objects for supporting the development of spatial reasoning skills in ways that engage youth. Because spatial thinking is associated with success in STEM disciplines, including mathematics, these activities are particularly important for students who are historically marginalized.

Keywords: *Spatial reasoning, geometry, out-of-school learning, STEM, curriculum.*

1. Introduction

There is strong evidence that concrete manipulatives—physical objects that offer learners sensory experiences of mathematical ideas—support learning when effectively integrated into learning activities (Carbonneau, Marley & Selig, 2013; Moyer, 2001). This case study focuses on the scale of 3D manipulatives, specifically examining whether human-scale versions have differential impacts on student engagement and spatial reasoning. In afterschool environments, fostering engagement is particularly important, as learners may disengage if an activity feels overly taxing or too similar to school.

We present activities from a co-design project that adapted in-school set of curricular materials focusing on mathematics, spatial reasoning, and making—for the use in an afterschool program at a community-based organization (CBO) in California, USA. TERC researchers supported facilitators through three hands-on 3-hour professional development sessions and on-site implementation during implementation in the fall of 2024.

Our study explores the affordances of differently scaled objects—cube puzzles made up of pieces created with one-inch linking cubes and one-foot boxes—for learners' participation in spatial activities and embodying spatial concepts and thinking. To assemble the linking cube pieces, youth had to use all three types of spatial reasoning skills: spatial perception, mental rotation, and spatial visualization (Linn & Peterson, 1985; Shriki & Patkin, 2021). In making the human-sized Soma Cube puzzle (one-foot boxes), youth embodied the spatial concepts of 3D rigid transformations: rotation and translation. In the large Soma Cube, youth step in and out of two perspectives: an embodied perspective as they move the puzzle pieces, and an observer stance, when they move back to observe someone else putting in a puzzle piece, offering a bird's eye view.

2. Literature review

Embodied learning theory posits that humans learn with their whole bodies, in experiences with tools and with other people, through which they develop concepts with analogous components in the world (Hall & Nemirovsky, 2012).

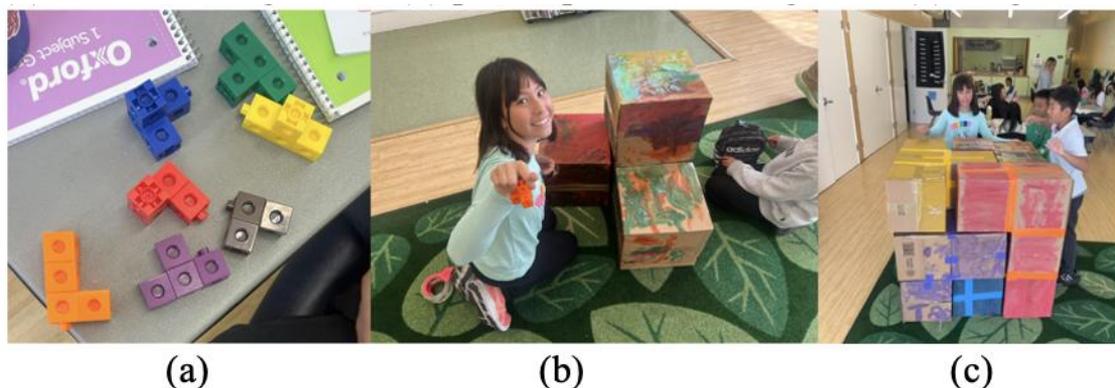
Opportunities to develop spatial reasoning are rare in the school mathematics curriculum, where one might expect them (see CCSS-Math to observe how rare they are). Yet spatial reasoning has been linked to success in STEM (Wai, et al., 2009) and in mathematics (Atit et al., 2022) and is recommended by NCTM for all grade levels (2000). Qualitative research has demonstrated that *making* activities provide rich opportunities to develop and use spatial reasoning (Ramey, K. E., Stevens, R., & Uttal, D. H., 2020). The out-of-school activities in this study have a strong focus on mental rotation, one category of spatial reasoning, (Shriki & Patkin, 2021), through students performing physical rotations, which have been identified as useful activities in developing mental rotation skills (e.g., Bruce, C. D., & Hawes, Z., 2015).

3. Design and methodology

We present activities from a co-design project to explore how an in-school set of curricular materials (MPACT) addressing mathematics, spatial reasoning, and making could be modified for use by informal learning facilitators in an afterschool program provided at a community-based organization (CBO) in California. TERC researchers provided hands-on professional development through three 3-hour trainings and on-site support during implementation in the fall of 2024.

Two pilots occurred of the human-sized puzzle activity. The first pilot took place at an established after-school homework club. Youth engaged in a variety of activities to develop spatial reasoning skills. They 3D printed their own designs in the first few sessions, and then in a session, they created a human-scale Soma Cube, made of 1'x1'x1' cardboard boxes. A Soma Cube is a 3x3x3 cube created from seven unique puzzle pieces, each made with 3- or 4- unit-cubes. During the first week of the Soma Cube activity, youth work individually or with tablemates with small 1-inch linking-cubes to create the seven puzzle pieces (Figure 1a). During the second week, teams of 3-4 youth each created one of the seven puzzle pieces with large cardboard boxes as unit cubes (Figure 1b). Then, they collaborated as a whole class to build the 3' x 3' x 3' Soma Cube puzzle (Figure 1c). A second pilot occurred at an additional site during their weeklong Spring break camp. These youth participated in the bookmark activity and Soma Cube activity over two days.

Figure 1. Pilot 1 (a) 1-inch linking cubes, (b) puzzle piece with large box, (c) large complete Soma Cube.



Researchers were present for key sessions with youth, and these sessions were photo- and video-recorded. Researchers asked youth about their methods for putting together the cube as they supported facilitation. The researchers wrote up descriptive notes after the session, and facilitators participated in post-session interviews. A larger research team watched the session videos together to identify key interactions around the Soma Cube construction, including verbal and non-verbal communication amongst the youth, and between youth and facilitator (Merriam & Tisdell, 2016, Lincoln & Guba, 1985).

4. Findings

Pilot 1

During the linking cubes activity, each youth successfully created a set of seven puzzle pieces, occasionally with hints or referencing peer's solutions. Most went on to find and assemble at least one full Soma Cube solution, although a few lost interest as they struggled to complete the cube.

Engagement was more sustained during the large cardboard cube assembly. Most youth remained engaged for over an hour as they constructed the human-sized individual puzzle pieces and the assembled Soma Cube. Working in pairs or trios, they used the small linking-cube versions as guides to build the cardboard pieces. Notably, youth who had previously disengaged were now actively participating with their peers. The scale of the cardboard pieces demanded collaboration. All 18 youth initially worked together to assemble the cardboard Soma Cube, following the lead from the one youth who had completed the smaller version. Despite the challenges of working in a large group, they completed the task successfully.

To promote deeper individual engagement, the facilitator later divided the class into small groups of 5-6, each taking turns to build the cube, ideally using alternative solutions—of which there are over 200. Compared to the previous week’s smaller linking cube activity, youth progressed further with the larger version. They also showed visible excitement, joy, celebrating their progress by cheering and playfully drumming on the puzzle pieces and the assembled cardboard Soma Cube.

Pilot 2

In the second pilot, Soma Cube building took place over two days. On the first day, youth created the puzzle pieces with linking cubes. Some quickly found the seven pieces, while others struggled to keep track of the design constraints and required more frequent hints from facilitators. Several youths successfully assembled the 3x3x3 cube after multiple attempts, demonstrating persistence. Others, however, gave up after their first failed attempt.

On the second day, pairs or trios collaborated to build each Soma Cube pieces with one cubic foot cardboard boxes. The large-scale pieces invited joyful make-believe interactions. Some youth used two of the large puzzle pieces to build a “lemonade stand” and ‘served’ lemonade from the T shaped puzzle piece. As a whole group, they spent 30 minutes trying to build the Soma Cube. While some youth persisted the entire time, others moved in and out of the activity. One or two youth chose to solve the cube with a smaller 3D printed version, intending the whole group to replicate their solution.

Meanwhile, those working with the large pieces attempted several iterations without success. Once the small-scale team arrived at a solution, they shared it with the group. Mapping the small solution to the large solution proved challenging and required extensive communication, including naming pieces for clarity. A particular linguistic mathematical challenge was communicating how pieces should be rotated. Words like turn and flip were used often, but which axis of rotation was not typically specified. This sometimes resulted in repeated requests, yelling, and occasionally, youth taking over rotating the piece. This group only completed one solution of the Soma Cube puzzle before the end of program time.

Figure 2. Pilot 2: Make-believe lemonade stand made from two puzzle pieces.



5. Discussion

Both groups of youth showed increased engagement with the Soma Cube activity with the human-scaled version of the puzzle. We conjecture that this has to do with the novelty of having such large manipulatives as well as the desire for and the space given to imaginative play during the activities.

Assembling the larger puzzle resulted in more active participation and more successful solutions. This may be in part because it is the second time youth engaged with the puzzle pieces, albeit at different scales. We conjecture that that the performative nature of such a collaborative project keeps most youth motivated and engaged for a longer time on the task.

Both groups struggled with communicating how to move the large cardboard pieces to complete the Soma Cube puzzle. This is not a surprise, given that we do not really teach 3D manipulations in elementary school, or really ever (e.g., Smith, 2005). Youth used everyday language to try to convey what they were envisioning in their heads: “Turn it this way.” In many cases, the object referred to by the word “it” was understood – as there was pointing or touching of the piece. However, it was not always clear which axis the piece should be turned over. Often the turn happened as a rotation around the z-axis, keeping the base of the piece oriented towards the ground. And just as often, the intended move required a rotation around a horizontal axis.

The learning activity along with the human-sized Soma Cube puzzle offered three key features as follows: 1) the scale encourages collaboration along with the associated communication work; 2) the size may encourage learners to mentally rotate an object due to the effort needed to physically rotate it; and 3) the manipulation of large size pieces may engage different embodied learning as it involves different muscle groups and movements for the different rigid transformations.

This case shows the promise of large-scale objects for supporting the development of spatial reasoning skills in ways that engage youth. Because spatial thinking is associated with success in STEM disciplines, including mathematics, these activities are particularly important for students who are historically marginalized.

6. Limitations

The informal learning settings for the activities we described constrained (perhaps productively so) the mathematical activities to those that would be actively engaging to the participants. Participants, even when they are at an age where their families choose their afterschool settings, have little incentive to engage in activities that they would not enjoy. It is possible that in a formal school setting that large scale manipulatives could support even more challenging spatial reasoning tasks along with the associated need for group mathematical communication. The participants’ general expectations of the kind of learning they would be engaging in could have also affected participants’ willingness to engage with what can sometimes be frustrating mathematical puzzles. Similarly, the research team did not have resources available to study the socio-mathematical norms of the programs. These norms, of course, can have a significant impact when participants (and their facilitators) engage in group math activities.

Further research with large scale manipulatives is certainly warranted given the joyful engagement and active spatial reasoning we have observed. Given the strong connection between spatial reasoning and successful STEM learning, the promise of building spatial reasoning skills and mathematical language, the promise of productive learning with large scale manipulatives is too great to ignore.

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