

STUDENT COMPETENCIES AND TEACHER KNOWLEDGE IN MATHEMATICS: A COMPARISON BETWEEN GERMANY AND BRAZIL

Nicole Steib¹, Sophia Ehlers¹, Stefan Krauss¹, Marcelo Firer², & Rita Santos Guimarães³

¹Department of Mathematics, University of Regensburg (Germany)

²Institute of Mathematics, Imecc - Unicamp (Brazil)

³Department of Mathematics, University of São Paulo (Brazil)

Abstract

International large-scale assessments such as the Program for International Student Assessment consistently reveal substantial country-level differences in mathematics achievement among 15-year-old students. But when do these gaps begin to form? Are they already present at school entry, or do they develop during the first years of schooling? To address these questions, we conducted a cross-cultural study combining student diagnostics and teacher knowledge assessment in Germany and Brazil. This approach (two populations: students and teachers; two countries: Germany and Brazil) aims to shed light on the timing and mechanisms behind international achievement gaps in mathematics. First, we assessed arithmetic competencies of first- and third-grade students in both countries ($N = 339$) using two diagnostic instruments tailored to the respective grades. These instruments capture number sense, basic operations, continuing sequences, and problem solving. Data were collected in Germany from $n = 75$ first graders (four classes in two schools) and $n = 93$ third graders (four classes in two schools), and in Brazil from $n = 84$ first graders (four classes in two schools) and $n = 87$ third graders (four classes in two schools). This allowed us to examine whether performance differences between countries already exist in Grade 1 or begin to emerge with formal schooling (i.e., between Grades 1 and 3). Second, we measured in both countries teachers' pedagogical content knowledge (PCK) — a key dimension of professional expertise that enables teachers to transform mathematical content into accessible learning experiences. We developed a new PCK test for primary school teachers based on several open-ended tasks. Items require analyzing specific student solutions, identifying misconceptions, and proposing instructional strategies. The PCK test was administered to a total of $N = 137$ participants, including $n = 63$ primary mathematics teachers in Brazil and $n = 74$ mathematics student teachers in Germany. Responses were evaluated using a scoring rubric that distinguishes levels of didactic reasoning, ranging from procedural to conceptual. Here, we discuss the theoretical foundations, methodological challenges, and first comparative results of both studies based on example items of the student and teacher tests (results for the full scales will be presented in the talk).

Keywords: *Pedagogical content knowledge, mathematics education, teacher education, cross-cultural comparison, assessment.*

1. Introduction

1.1. International achievement gaps in mathematics

International large-scale studies have consistently documented substantial differences in mathematics achievement across countries, for instance, between Germany and Brazil: For example, in the Program for International Student Assessment (PISA) 2022, German ninth graders scored 475 on average compared to a score of 379 in Brazil. This difference is roughly equivalent to one standard deviation (Lewalter et al., 2023; Organisation for Economic Co-operation and Development [OECD], 2023). Concerning fourth graders, the gap is similarly pronounced in the 2023 Trends in International Mathematics and Science Study (TIMSS), where Germany reached a mean score of 524 points but first-time-participant Brazil scored 400 points, which again corresponds to a difference of one standard deviation (von Davier et al., 2024).

Despite the robustness of such findings for students between the ages of 10 and 15 years, relatively little is known about when these gaps first emerge or how they develop across the early years of schooling. It remains unclear whether the observed differences are already present at school entry — suggesting pre-school or non-school factors are the primary drivers — or whether they emerge or widen during the first years of formal instruction at elementary school.

1.2. Mirrored digit writing

One aspect of early arithmetic skills is the correct notation of numbers. So-called mirrored digit writing — the horizontal reversal of individual digit forms during handwriting — is a well-documented phenomenon in early childhood (e.g., Fig. 3). Around age 5, children enter a developmental phase in which they can recognize digits but display uncertainty about their spatial orientation (Fischer & Luxembourger, 2018). The reversal tendency declines rapidly after age 6. Fischer and Koch (2016) found reversal rates (i.e., the percentage of digits written mirrored) of around 9% at ages 6 to 7, and Goebel et al. (2019) reported comparable patterns in first graders.

A leading theoretical account attributes this to *mirror generalization* (Fischer & Luxembourger, 2018). The visual system initially encodes objects independently of their left–right orientation, an adaptive mechanism useful for object recognition in everyday life. When children reproduce a digit from memory, orientation information may be lost, rendering the mirrored form equally valid. Children then tend to resolve this ambiguity in favor of a rightward orientation, consistent with the implicit directionality of the left-to-right writing system. Fischer (2025) found no association with slower arithmetic performance, and children who systematically reversed certain digits even showed slightly better oral addition scores, suggesting an active and rule-governed orientation strategy rather than a deficit.

To date, research on this phenomenon has focused almost exclusively on Western European samples, leaving open the question of whether developmental trajectories and prevalence rates differ across educational contexts. Among others, the present study addresses this gap.

1.3. Teacher knowledge

One possible explanation for cross-national differences in student outcomes (see Section 1.1.) is the quality of mathematics instruction, which is closely linked to teachers' professional knowledge. Shulman (1986) introduced the concept of *pedagogical content knowledge* (PCK) to describe subject-specific knowledge that enables teachers to make content accessible to learners (e.g., knowledge of typical student difficulties or misconceptions and of helpful visualizations and representations of content) (Ball et al., 2008; Krauss et al., 2020).

Empirical evidence supports the predictive relevance of PCK for student learning. In Germany, for instance, the Cognitive Activation in the Classroom study (Kunter et al., 2013) demonstrated that mathematics teachers' PCK is highly predictive both for instructional quality of mathematics lessons and for students' learning gains (Baumert et al., 2010; Simböck et al., in press). These results suggest that possible differences in teacher knowledge could partially account for the observed achievement gap described in Section 1.1., motivating the inclusion of a PCK assessment when trying to examine the discrepancy.

2. Objective

In the present proceedings paper, we shed light on the emergence of the aforementioned achievement gaps in mathematics between students in Germany and Brazil. In particular, we take a first glance at the following research questions based on example items:

1. Are differences in performance of German and Brazilian students already observable at the beginning of formal elementary schooling, specifically in the first grade?
Should substantial disparities be evident at this stage, this would suggest that factors preceding or independent of formal instruction play a significant role in explaining this. Conversely, if differences in first grade are small or nonexistent but widen until third grade, this might imply that the schooling process itself contributes to the divergence in achievement.
2. Do German pre-service teachers and Brazilian practicing teachers demonstrate PCK levels that align with the better student outcomes in Germany compared to Brazil?
By comparing PCK levels among German and Brazilian educators, we aim to search for possible reasons for variations in student performance.
3. Is the prevalence of mirrored digit writing the same in Germany and Brazil and is it linked to arithmetic achievement?

When finally analyzed, this study will offer insights into the timing and potential mechanisms behind international achievement gaps, with implications for early mathematics education and teacher training. A more detailed analysis (i.e., going beyond example items) that includes results for the full scales of the student and teacher tests will be presented in the talk.

3. Method

To investigate the research questions, we employed a two-component design, assessing mathematical competencies of primary school students as well as the PCK of current (Brazil) and future (Germany) teachers. The instruments for students as well as teachers were specifically designed for this cross-national study, developed in collaboration of our Brazilian-German research team.

3.1. Student test: Arithmetic competencies

The arithmetic competencies of German and Brazilian students were measured using grade-specific tests. Both instruments, for first- and third-grade students, target four domains: number sense, basic operations, continuing sequences, and problem solving. To enable cross-sectional comparisons of competencies between different grades, four items were included in both the Grade 1 and Grade 3 instruments in identical form, serving as anchor items. One of these anchor items (“continue the sequence”) can be seen in Figure 1.

Figure 1. Anchor item “continue the sequence” in the assessment of students’ arithmetic competence (Brazilian version).

COMPLETE AS SEQUÊNCIAS.

19	17	15							
----	----	----	--	--	--	--	--	--	--

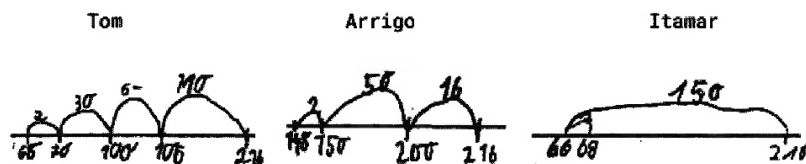
Regarding this item, the numbers written in each cell were recorded for each child and it was coded whether the sequence was continued correctly or not (further example items including their coding procedures will be presented in the talk). In addition, we coded the presence of the mirroring phenomenon in a binary way (i.e., whether at least one digit in a student answer was mirrored).

3.2. Teacher test: PCK

The test of Brazilian teachers’ and German student teachers’ PCK consists of six open-ended items that require participants to engage in reasoning that characterizes effective teaching practice. For instance, the PCK items covered knowledge on how to explain mathematical content, on how to deal with typical student errors, or on how to analyze the reasoning behind various student answers to a problem. Figure 2 displays a PCK item where teachers had to reproduce subtracting strategies of students on the number line for a further subtraction task.

Figure 2. Anchor item “subtraction strategies” in the assessment of teachers’ Pedagogical Content Knowledge (Brazilian version). [English translation: “The figures below show, on a number line, three ways to perform the subtraction 216–148. Each of these methods represents a different type of reasoning. Solve the subtraction 317–198 using each of the three methods of reasoning above.”]

As figuras abaixo representam, numa reta numérica, três modos de efetuar a subtração 216–148. Cada um destes modos representa um tipo diferente de raciocínio.



Resolva a subtração 317–198 seguindo cada um dos três tipos de raciocínios acima.

3.3. Sample

3.3.1. Students. Data were collected in Germany from $n = 75$ first graders and $n = 93$ third graders, and in Brazil from $n = 84$ first graders and $n = 87$ third graders. Regarding each of the four subsamples, students stem from four classes of two schools. The total sample size was $N = 339$ students.

3.3.2. Teachers and Student Teachers. The instrument was administered to $n = 74$ pre-service mathematics teachers in Germany and $n = 63$ in-service mathematics teachers in Brazil, yielding a total sample size of $N = 137$. In 2027, as the project progresses, we will also examine the corresponding groups in the other country (i.e., student teachers in Brazil and practicing teachers in Germany).

4. Results

All statistical analyses were performed using SPSS.

4.1. Student test: Arithmetic competencies

For the item displayed in Figure 1 we obtained the results shown in Table 1. While 19% of Brazilian first graders solved the item shown in Figure 1 correctly, 68% of their German peers did the same. In both countries, the third-grade students solved the item correctly more often than their younger peers, with about 41% in Brazil and ca. 83% in Germany doing so. What is particularly striking here is that German first-grade students correctly solved the item even more often than their older (third grade) Brazilian counterparts.

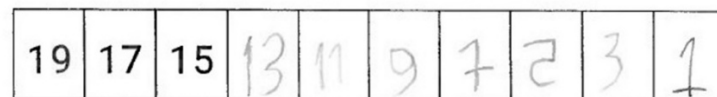
Table 1. Solution rates for the item shown in Figure 1 and prevalence of at least one mirrored digit.

	Brazil		Germany	
	Grade 1	Grade 3	Grade 1	Grade 3
Item correct	19.0%	41.2%	68.0%	82.8%
Prevalence of digit mirroring	26.2%	8.3%	0%	0%

Interestingly and surprisingly, the mirrored writing of digits (for an example answer of a Brazilian first grader, see Figure 3) occurred only among Brazilian students, although the prevalence decreased markedly between first and third grade. In contrast to previous findings (see Section 1.1.), the prevalence of mirrored digit writing in our study appears to be associated with lower performance.

Figure 3. A Brazilian first grader's answer to the anchor item: "continue the sequence." The digit "5" is mirrored.

COMPLETE AS SEQUÊNCIAS.

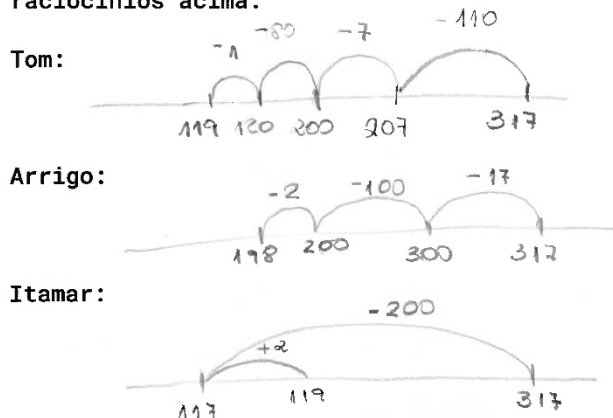


4.2. Teacher test: PCK

The analysis of the answers from Brazilian teachers and German student teachers is work in progress (we can present descriptives in the talk). A first glance at our data indicates that German student teachers even seem to outperform practicing teachers in Brazil despite not having finished their teacher training yet. Figure 4 displays the answer from one Brazilian teacher reproducing the students' subtraction strategy for the new subtraction task 317-198 correctly.

Figure 4. A Brazilian teacher's answer to the question in Figure 2.

Resolva a subtração 317-198 seguindo cada um dos três tipos de raciocínios acima.



5. Discussion

In sum, and when considering the example items, the differences of teachers align with the performance differences of students. The results of this study reveal a striking pattern: German first graders already outperformed Brazilian third graders in an exemplary foundational mathematical skill. In addition, the differences observed in first grade indicate that factors preceding formal elementary classroom instruction play a role in the emergence of this pattern. However, the persistence and even amplification of these gaps by third grade imply that school-related factors, including instructional quality or teacher competencies, may further contribute to the divergence in student outcomes.

Indeed, a parallel pattern emerges in the assessment of teachers' PCK. Remarkably, German pre-service teachers who have yet to gain practical teaching experience outperformed Brazilian in-service teachers on the PCK test. Given that PCK is a critical determinant of instructional effectiveness, these findings imply that differences in teacher preparation could at least partially explain the observed achievement gaps in international assessments such as the TIMSS and PISA.

While the current analysis is based on single-item results, a first glance at the other items in our data reveals similar findings. Therefore, the items displayed in Figures 1 and 2 are representative of the corresponding full scales. Results of other items as well as of the complete scales will be presented in the talk.

Acknowledgments

This work was partially supported by Fapesp, grant 2022/06882-4.

References

- Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching. *Journal of Teacher Education*, 59(5), 389–407.
- Baumert, J. & Kunter, M. & Blum, W. & Brunner, M. & Voss (Dubberke), Tamar & J., Alexander & Klusmann, U. & Krauss, S. & Neubrand, M. & Tsai, Y.-M. (2010). Teachers' Mathematical Knowledge, Cognitive Activation in the Classroom, and Student Progress. *American Educational Research Journal*, 47, 133-180. Doi: 10.3102/0002831209345157
- Fischer, J. P. (2025). *Children's mirror writing: Applying a theory*. HAL Open Science. <https://hal.science/hal-05086002>.
- Fischer, J. P., & Koch, A. M. (2016). Mirror writing in typically developing children: A first longitudinal study. *Cognitive Development*, 38, 114–124. <https://doi.org/10.1016/j.cogdev.2016.02.005>
- Fischer, J.-P., & Luxembourger, C. (2018). A synoptic and theoretical account of character (digits and capital letters) reversal in writings by typically developing children. *Education Sciences*, 8(3), Article 137. <https://doi.org/10.3390/educsci8030137>
- Goebel, S. M., Clayton, F. J., Khanijou, N., Diaz, M. R., & Weiers, H. (2019). *Mirror writing of digits and its relation to mathematical performance in primary school children*. OSF Home. <https://osf.io/bxv46>
- Krauss, S., Bruckmaier, G., Lindl, A., Hilbert, S., Binder, K., Steib, N., & Blum, W. (2020). Competence as a continuum in the COACTIV study: The “cascade model.” *ZDM*, 52(2), 311–327. <https://doi.org/10.1007/s11858-020-01151-z>
- Kunter, M., Baumert, J., Blum, W., Klusmann, U., Krauss, S., & Neubrand, M. (2013). *Cognitive activation in the mathematics classroom and professional competence of teachers*. Springer US. <https://doi.org/10.1007/978-1-4614-5149-5>
- Lewalter, D., Diedrich, J., Goldhammer, F., Köller, O., & Reiss, K. (2023). *PISA 2022*. Waxmann Verlag GmbH. <https://doi.org/10.31244/9783830998488>
- Organisation for Economic Co-operation and Development. (2023). *PISA 2022 Technical Report*. OECD Publishing. <https://doi.org/10.1787/01820d6d-en>
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14.
- Simböck, L., Lindl, A., & Krauss, S. (in press). Professionswissen und Unterrichtsqualität von Mathematiklehrkräften – erste Befunde aus FALKO-PV. *Beiträge zum Mathematikunterricht 2026*. WTM.
- von Davier, M., Kennedy, A., Reynolds, K., Fishbein, B., Khorramdel, L., Aldrich, C., Bookbinder, A., Bezirhan, U., & Yin, L. (2024). *TIMSS 2023 international results in mathematics and science*. Boston College, TIMSS & PIRLS International Study Center. <https://doi.org/10.6017/lse.tpisc.timss.rs6460>