

FORCE MEASUREMENT FOR DEMONSTRATION OF NON-LINEAR ASPECTS OF HYDRODYNAMICS FOR ELEMENTARY AND SECONDARY SCHOOL STUDENTS

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Abstract

Hydrodynamics represents an attractive and visually engaging branch of physics that can be effectively used to popularize science among students. At the University of West Bohemia in Pilsen, demonstration experiments have been developed to introduce the fundamental principles of aerodynamics through force measurements in a wind tunnel. These experiments allow students themselves to participate actively in the measurement process. A representative example involves measuring drag forces acting on Lego model. Students control the tunnel velocity and angle of attack, thereby experiencing firsthand the quadratic dependence of drag on velocity. The results show the steep growth of aerodynamic forces with increasing speed, highlighting the energetic demands of high-speed transportation. Further demonstrations emphasize the nonlinear nature of fluid flows. For instance, splitting an obstacle into two blocks does not lead to a simple linear superposition of forces; instead, the measured drag depends strongly on the separation distance. The presented experiments are safe, time-efficient, and suitable for interactive engagement during school visits. They provide children with the opportunity to manipulate experimental parameters, observe real-time data, and even design their own obstacles. Such activities foster curiosity, abstract thinking, and appreciation of the complexity and beauty of the nature.

Keywords: Drag force, bluff body, wind tunnel, demonstration experiment, excursion.

1. Introduction

Hydrodynamics or aerodynamics does not belong to deeply studied subjects within the elementary- and secondary school. However, we are submerged in a fluid – air, and significant part of our lives has to do with water (swimming, drinking) or other flowing liquids (blood, oils, fuels...). Instead, the *hydrostatics* is used to be taught at elementary schools, probably, because it contains simple formulae, which is easy to test. We believe, that the physics of flowing fluids reserves attention as well (or maybe even more than the simple linear hydrostatics) not only because it is interesting and beautiful (an often neglected but important aspect of nature), but also, because it shows one important aspect of many complex physical systems – the non-linearity. The ideas of deterministic chaos, self-organization, memory of a system or fractality and multiscality apply in hydrodynamics and they are observable without a need of too high mathematics and abstract thinking. Instead, hydrodynamics may motivate students for abstract thinking and mathematics due to its aesthetics and usefulness at the same time.

At the last END conference, we demonstrated the time-symmetry breaking on a simple example of fluid jet (List, 1982) by using a rheoscopic fluid (Borrero-Echeverry, Crowley, & Riddick, 2018) within a simple model, which the children can easily touch (Duda et al., 2024; Yanovych et al., 2019). At the current END conference, we plan to present other aspects of non-linearity in hydromechanics: the force.

1.1. Aerodynamic drag force

The aerodynamic drag force is dependent on flow velocity u , fluid density ρ and obstacle cross-sectional area S as

$$F_D = \frac{1}{2} \cdot \rho \cdot S \cdot u^2 \cdot C_D \quad (1)$$

where the drag coefficient C_D represents all the interesting effects, which are not directly derivable (Wu, Ma, & Zhou, 2006). Note, that C_D itself is dependent on velocity, on the shape details of the body, on the roughness of the surface, and also on the turbulence level and structure of incoming flow.

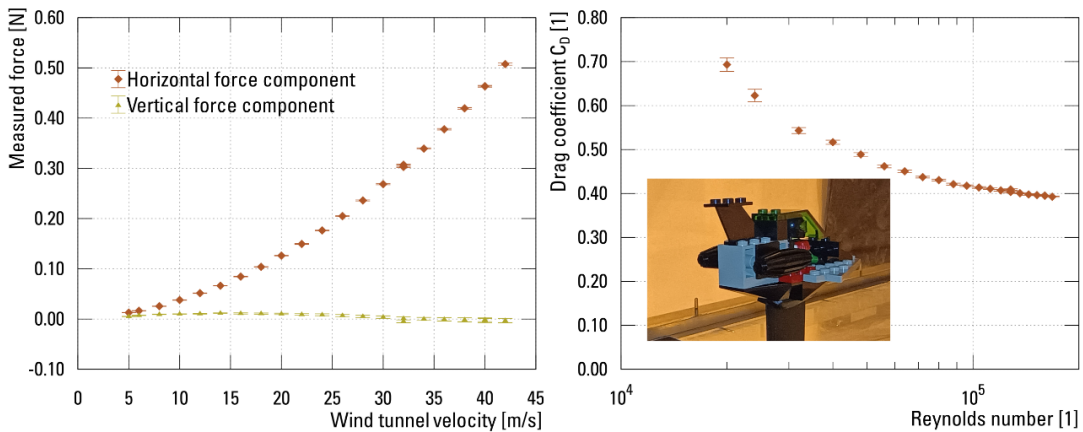
This formula is standardly studied at secondary schools, but usually without some deeper discussion. First important consequence of the formula (1) is, that the aerodynamic force can be neglected for usual human-scale conditions (e.g., walking or riding a bicycle), but it becomes very important at high-speed transportation (e.g., cars, trains or airplanes). When riding on a bicycle, what scares you, is a hill, not a counter-wind. On the other hand, when driving inside a car on a high-speed highway or motorway, it can easily happen, that it is needed to push acceleration even when going *downhill* to just keep constant velocity! The quadratic dependence on velocity says, that twice faster means four-times higher aerodynamic forces. And even worse, the power needed to overpass the drag force is:

$$\dot{W} = F \cdot v \sim v^3 \quad (2)$$

(when we consider that the fluid velocity u is equal to the body velocity v .) This is a very steep scaling ensuring, that during walking, you do not know about air around you, but the same air evaporates a meteorite within a fraction of second. Increasing the velocity by 1 % costs 3 % more power. This is the reason, why the so-called sport-cars are so embarrassing: for example: Bugatti Chiron has power 1.1 MW, but the maximal velocity is *only* 490 km/h. When comparing with some *normal* car, e.g., Ford C-Max has power 85 kW and it reaches 160 km/h without any problem (tested by author). Additionally, Ford can transport up to 7 persons (Bugatti 2) and also the price is different (~2 M€ vs. ~10 k€). The high-speed becomes very expensive above velocities about ~30 m/s (108 km/h). We believe, that the children should know this, because later they grow up, get a driving license and without such knowledge, they will just ineffectively burn tons of fuel until the entire planet turns into a desert.

2. Example force measurement during the children's visit at university

Figure 1. The example of force data measured by children. The obstacle (which can be anything fitting into wind tunnel and connectable to the tensometer) is the model no. 6813 “Galactic Chief” from the company Lego. The only modification is the replacement of 2×1 hinge for cabin opening by a solid standard brick 2×1 to avoid cabin opening during measurement (see the inset photograph). Note that the ship has to be connected to the gauge by using a stand (black part under the ship in the inset photograph).



Children from elementary or secondary schools in the city of Pilsen regularly come to visit the University of West Bohemia. They look at professional scientific equipment, they admire the nice campus and they are happy, that there is a different activity than just sit at school at least for one day. If they come into our laboratory, we show them the wind tunnel (Yanovych et al., 2019), which can reach the velocity of 90 m/s (with empty test section). While we prefer measurement by using PIV (Duda & Yanovych, 2024) for scientific research, this method is too complicated for a fast overview (and also dangerous due to the pulsed laser). On the other hand, the force measurement is quite straight-forward: we mount an obstacle to the commercial force/torque gauge F6D45 from the company ME-Meßsysteme GmbH and by varying the

wind tunnel velocity or angle of attack (or different parameter), an interesting dependence is measured within few minutes, see Figure 1. Mainly, it can be measured by the visitors: the first selected child controls the wind tunnel velocity, the second one controls the angle of attack and the third one observes the instantaneous force and decides, when to save the data point. The children are usually excited from such a responsible task and we have not observed any troubles during this procedure.

Because the first author is just an overgrown child, he likes Lego spaceships. That is a great kind of obstacles familiar for children-age visitors – the geometry is complex and various effects can be easily demonstrated. Figure 1 shows an example of a such measurement: the obstacle is a vessel no. 6813 from *Space Police II* from the year 1993. The steep increase of force is clearly visible in the plot. The custom-made program displays the live data during measurement, hence the visitors observe in real time, as the force grows, then it grows more and more and faster and faster, which, we believe, is the experience better than just a formula.

The dimensionless drag coefficient C_D is calculated by dividing the measured force by fluid density, cross-sectional area (which can be a nice geometric exercise for visitors, if they have enough time) and mainly by the square of velocity. We see, that C_D decreases with Reynolds number (“dimensionless velocity”, $Re = u \cdot L / \nu$, L is the obstacle width, 8 studs, i.e., 64 mm for the model 6813, ν is the kinematic viscosity of air, $1.5 \cdot 10^{-5} \text{ m}^2/\text{s}$). Scientific studies usually display this kind of results (C_D as a function of Re) and the decrease of drag coefficient is an often-reported property. However, the force increases more dramatically, so steep, that some variations in C_D are barely visible.

3. Non-linearity of the flow

Significant part of the success of science and technology in 19th and 20th century is caused by practical use of the linear decompositions. Many problems can be separated into individual simpler subproblems, they are solved separately and then connected back. For example, the heat conduction in solid material, evaluation of forces from different sources or passive electrical circuits can be solved by using the “*divide et impera*” approach. However, the flowing fluid is a different story. At the previous END conference (Duda et al., 2024) we demonstrated, that inverting the direction of velocity does not lead to inverted solution (velocity field); it produces a completely different solution!

Motion of incompressible liquid is described by the famous Navier-Stokes equations:

$$\frac{d\vec{u}}{dt} = \frac{\partial \vec{u}}{\partial t} + \underbrace{(\vec{u} \cdot \nabla) \vec{u}}_{\text{Advection}} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u} + \vec{g} \quad (3)$$

$$\nabla \cdot \vec{u} = 0$$

Except the advection term, everything else is linear and can be ruled by using the *divide et impera* scheme. The advection term cannot be neglected – it is not connected with some force or with some material property. It is caused by the transformation from the reference frame of individual fluid particles (where the physical forces act) into the laboratory reference frame, which is needed to describe the flow in an objective way. Hence, it is principal term.

Although children at elementary or secondary schools are not familiar with the vector fields, neither with differential operators, the non-linearity can make an analogy to the decomposition of $(a + b)^2 = a^2 + 2ab + b^2$, which belongs to the elementary mathematics. Similarly, when considering decomposition of $u = a + b$, the advection term would decompose into

$$((a + b) \cdot \nabla) (a + b) = (a \cdot \nabla) a + (a \cdot \nabla) b + (b \cdot \nabla) a + (b \cdot \nabla) b \quad (4)$$

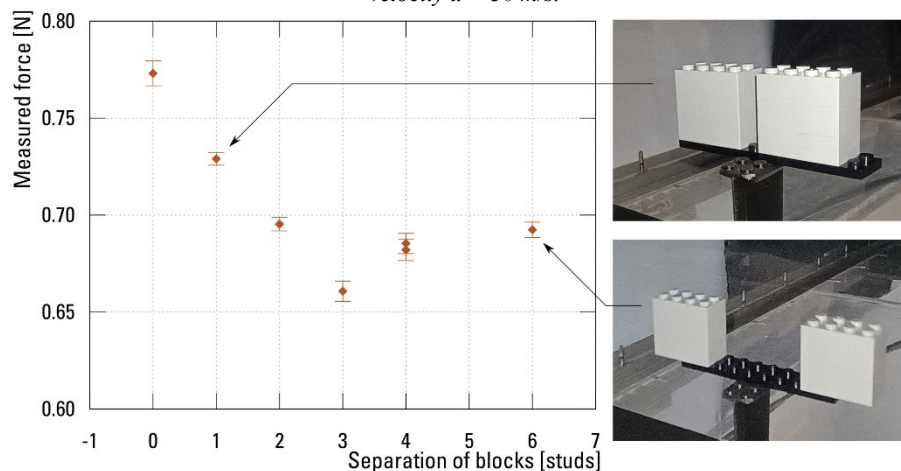
where the first and last term are ok, they would belong to the corresponding Navier Stokes equations for a and for b . But where to put the mixed terms $(a \cdot \nabla) b$ and $(b \cdot \nabla) a$? For example, if we would like to decompose the velocity field mentioned above (*Galactic chief* connected to a stand) into the flow belonging to the vessel alone and to the contribution of the stand (which is needed to be there from the practical purposes), then we can measure the effect of the field u (the vessel + the stand), then measure the effect of the field b (the stand alone). But, the field $a = u - b$ (expected to belong to the vessel alone) CANNOT fulfil Navier-Stokes equations! The flow around it will be completely different. Similarly, it makes no sense to solve the flow fields around individual bricks and then connect them together as we can do with the bricks.

3.1. Example on summation of forces

Straight-forward example is presented to the children: we have two walls, again made of Lego bricks. These walls are placed side by side facing the flow. When the walls touch each other, then there is some force acting on them. But this force is different from twice the force acting on each of them separately.

In the simple experiment, we change the distance between the blocks from 0 (touching, effectively single wall) up to 6 studs (1.5 × the single block width) and observe the dependence of force on this separation, see Figure 2. When the blocks are far from each other, the value can be considered as linear superposition. Note, that the measured values at separations 4 and 6 studs are similar. When it is effectively a single obstacle (0 separation), then the reported drag force is highest because the wake width (Duda, Yanovych, & Uruba, 2021) is largest (we can estimate, that it is proportional to the width of the entire obstacle). If there appears a gap under the stagnation point, the place with otherwise the highest pressure is ventilated. The fluid at the body front is not so accelerated to sides and thus the wake width decreases. This effect “overshoots” the value observed for distant obstacles, and at separation of 3 studs we observe a minimum. The minimal value is by 5 % smaller than the drag force of separated obstacles and by 15 % smaller than the force acting on connected obstacles.

Figure 2. Another example of force measurement. This case highlights the non-linearity of flows, i.e. the impossibility to use the “divide et impera” scheme - a pair of walls is separated by variable distance and the resulting force is **not** a sum of forces acting on left and right block. Measurement performed at constant velocity $u = 30$ m/s.



4. Conclusion

The university hydrodynamic research is presented to visitors from elementary or secondary schools. A relatively simple experiments in wind tunnel allow to demonstrate some of the basic characteristics of aerodynamics by using the drag force measurement. The drag force acting on any bluff body grows with velocity approximately as $F_D \sim u^2$, which is quite a steep scaling which humans do not have direct experience with, although it becomes significant for the high-speed transportation (over ~ 30 m/s). This effect is demonstrated on a small Lego model of a spaceship no. 6813 “Galactic Chief”, but it can be anything else.

Another interesting and important issue is the non-linearity of flow phenomena. The velocity field cannot be decomposed into parts that all fulfil the Navier-Stokes equations, therefore they cannot be explored separately. We demonstrate this by measuring drag force acting on a pair of parallel blocks with modified separation. The force dependence on separation displays decrease with overshoot followed by a plateau.

These simple experiments are relatively fast to perform, which is ideal for visits from elementary and secondary schools, when the children do not have much time (and patience). The visitors can put hands on the experiments in terms of controlling the wind tunnel speed, or, if there is enough time, they can construct their own obstacle by using the Lego and then compare competing constructions (e.g., one task can be to construct a shape, which contains a minifigure, but has minimal drag).

Note that the goal of this activity is not to pass some exam or increase some score of the students’ knowledge, but rather to *motivate* them for future study of the beauty of nature. The interesting effects are often hidden behind a difficult mathematics, but on the example with flow decomposition we demonstrated the effects of non-linearity directly on a behaviour of aerodynamic force. To know, that the mathematical characteristics has some direct consequence on behaviour of a physical object, helps the students to accept the need of mathematical studies for their applicability within the technical civilization. The described methods combine observation of a demonstration with possible controlling of the experiments, or, in the case of enough time, with creative production of a custom-made obstacle and characterizing its aerodynamic properties.

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References

- Borrero-Echeverry, D., Crowley, C. J., & Riddick, T. P. (2018). Rheoscopic fluids in a post-Kalliroscope world. *Physics of Fluids*, 30(8), 087103.
- Duda, D., Tomášková, T., Yanovych, V., Krotký, J., & Uruba, V. (2024). Demonstration of time-symmetry breaking in a jet flow. *Education and New Developments 2024 – Volume 2*. Lisbon: inScience Press.
- Duda, D., & Yanovych, V. (2024). Interaction of stream-wise vortices generated by swirler grid. *Physics of Fluids*, 36(5).
- Duda, D., Yanovych, V., & Uruba, V. (2021). Wake width: Discussion of several methods how to estimate it by using measured experimental data. *Energies*, 14(15), 4712.
- List, E. J. (1982). Turbulent jets and plumes. *Annual Review of Fluid Mechanics*, 14, 189–212.
- Wu, J. Z., Ma, H. Y. & Zhou, M. D. (2006). *Vorticity and Vortex Dynamics*. Springer, Heidelberg.
- Yanovych, V., Duda, D., Horáček, V., & Uruba, V. (2019). Research of a wind tunnel parameters by means of cross-section analysis of air flow profiles. *AIP Conference Proceedings*, 2189(020024).