

THE MEDIATING ROLE OF MISCONCEPTIONS IN BLENDED LEARNING'S IMPACT ON 3D TRIGONOMETRY ACHIEVEMENT: A STRUCTURAL EQUATION MODELLING STUDY

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Abstract

This study examines the mediating roles of conceptual understanding (CU) and misconceptions (ME) in the relationship between blended learning (BL) and academic achievement (ACH) in Grade 12 three-dimensional (3D) trigonometry. Grounded in social constructivism, an explanatory sequential mixed-methods design was employed, integrating structural equation modelling (SEM) with qualitative interviews and classroom observations. Data were drawn from 381 learner scripts and a focal sub-sample of 30 students and three teachers in South African public schools situated within the Chris Hani West District. Quantitative results confirm that BL positively predicts conceptual understanding ($\beta = 0.32$) and academic achievement ($\beta = 0.25$), while negatively predicting misconception ($\beta = -0.21$). Conceptual understanding reduces misconceptions ($\beta = -0.40$) and enhances academic achievement ($\beta = 0.35$), with misconceptions negatively impacting performance ($\beta = -0.28$). Bootstrapped mediation analysis identifies significant indirect effects of blended learning on academic achievement through conceptual understanding ($\beta = 0.11$) and misconceptions ($\beta = 0.06$), and via a serial pathway via conceptual understanding $\rightarrow$ misconceptions $\rightarrow$ academic achievement ($\beta = 0.10$). Conceptual errors accounted for 65.70% of learner scripts. Qualitative triangulation confirms that dynamic visualisations, dialogic tasks, and formative feedback are most effective in disrupting persistent error patterns. The study argues that blended learning, when strategically designed to target misconceptions, improves conceptual mastery and, by reducing errors, indirectly boosts achievement. Findings offer actionable instructional design principles for STEM education in resource-constrained settings.

Keywords: *Blended learning, misconceptions, 3D trigonometry, mathematics achievement, structural equation modelling.*

1. Introduction

Mathematics education in South Africa confronts deep-rooted systemic challenges, including chronic underresourcing, teacher capacity deficits, and gaps in curriculum implementation, that collectively depress learning outcomes across secondary schooling (Maqoqa & Saziwa, 2025; Muyambi & Ahiaku, 2025). These pressures are most acutely felt in spatially demanding topics, of which three-dimensional (3D) trigonometry is among the most cognitively taxing for Grade 12 learners. The topic demands simultaneous facility with spatial reasoning, diagram interpretation, and the application of trigonometric ratios across planes and solids, competencies that learners frequently lack (Hamzic & Takaci, 2017).

Blended learning, the systematic integration of digital instructional tools with face-to-face pedagogy, has attracted growing interest as a mechanism for addressing such conceptual difficulties (Baidoo & Luneta, 2024). Dynamic visualisation platforms such as GeoGebra offer spatial scaffolding that static textbook diagrams cannot provide, while asynchronous e-quizzes enable formative feedback at scale (Bayaga, 2024). Despite this potential, the field lacks rigorous empirical modelling of precisely how blended learning quality translates into achievement gains. Specifically, the mediating roles of conceptual understanding and misconceptions in this relationship remain undertheorised and underquantified.

This study addresses that gap by testing a theoretically grounded structural model in which BL influences ACH both directly and indirectly through CU and ME. Three hypotheses organise the inquiry: (H₁) BL positively predicts CU and ACH while negatively predicting ME; (H₂) CU reduces ME and enhances ACH; and (H₃) ME mediates the BL-ACH relationship. By integrating SEM with qualitative classroom data, the study contributes both statistical rigour and contextual depth to the blended learning literature.

2. Literature review

2.1. Blended learning in mathematics education

Blended learning offers a flexible instructional architecture that allows teachers to combine synchronous, face-to-face interaction with asynchronous digital engagement. In mathematics, this dual modality enables learners to revisit challenging concepts at their own pace while still benefiting from teacher-mediated dialogue (Bayaga, 2024). Juandi et al. (2025) demonstrate that blended delivery significantly improves secondary learners' conceptual understanding of mathematics compared to traditional instruction, particularly when digital tasks are aligned with specific conceptual goals rather than used as mere supplements. In the South African context, Baidoo and Luneta (2024) document the specific affordances of blended learning for 3D trigonometry, noting that GeoGebra-based lessons dramatically improve learners' ability to identify angle relationships in pyramids and prisms. This improvement is attributed not to technology per se, but to the quality of instructional design embedded within blended environments, a distinction that underscores the centrality of BL as a construct in the present study.

2.2. Misconceptions and error analysis in trigonometry

Misconceptions in mathematics are not random errors; they are systematic, often internally consistent cognitive structures that resist correction without targeted intervention (Baidoo & Luneta, 2024). In trigonometry, learners persistently conflate 2D and 3D angle properties, misapply trigonometric ratios to three-dimensional solids, and struggle to project spatial configurations onto two-dimensional representations (Juandi et al., 2025). Skemp's (1978) distinction between relational and instrumental understanding is instructive here: learners operating instrumentally may execute trigonometric procedures correctly in familiar contexts but collapse when spatial demands increase, precisely because their underlying conceptual schema is brittle. Error analysis, as a diagnostic methodology, offers a structured means of identifying and classifying misconception patterns in learner work (Baidoo & Luneta, 2024). Applied to 3D trigonometry scripts, it reveals that conceptual errors, rather than procedural calculation mistakes, are the dominant impediment to achievement, with direct implications for instructional targeting.

2.3. Constructivism as a theoretical framework

This study is grounded in Vygotsky's social constructivism (Vygotsky & Cole, 2018), which holds that conceptual development occurs through mediated, socially scaffolded interaction. Blended learning environments instantiate this framework by providing multiple, layered mediating tools: dynamic digital representations, peer discourse tasks, and formative feedback systems. Dahl and Mørch (2025) argue that constructivist mathematics pedagogy requires learners to actively confront cognitive conflict; blended environments, when well-designed, systematically engineer such conflict through peer comparison activities and immediate error notification. The present study positions BL as the instructional operationalisation of constructivist design quality.

3. Methodology

3.1. Research design

An explanatory sequential mixed-methods design (Haynes-Brown, 2025) was employed. In Phase 1, SEM was used to quantify and test hypothesised structural relationships. In Phase 2, qualitative data from semi-structured interviews and classroom observations were thematically analysed (Haynes-Brown, 2025) to contextualise and explain quantitative outcomes. This sequencing ensures that qualitative inquiry is driven by, and responsive to, specific patterns emerging from the structural model.

3.2. Participants and setting

Participants were Grade 12 mathematics learners ($N = 381$) drawn from six purposively selected public high schools in the Chris Hani West District, Eastern Cape, South Africa. Schools were selected to represent a range of resource levels while maintaining curriculum comparability. A focal sub-sample of 30 learners, stratified by achievement quartile, and three mathematics teachers participated in qualitative data collection. Prior term scores served as covariates to control for baseline differences.

3.3. Measures

Four latent constructs were operationalised. BL was assessed through 20 structured classroom observations, teacher instructional logs, and learner surveys ($\alpha = 0.86$), with indicators capturing visualisation tool use, digital task alignment, and feedback quality. CU was measured via a 15-item validated test assessing angle definitions, spatial reasoning, and diagram interpretation ($\alpha = 0.88$). ME was

operationalised through error-coded learner scripts using a four-category rubric (conceptual, transformation/representation, procedural, encoding), with high inter-rater reliability ($\kappa = 0.82$). ACH was measured by a standardised 3D trigonometry examination aligned with the South African CAPS curriculum ($\alpha = 0.90$).

3.4. Analytical strategy

Quantitative data were analysed in Mplus 8.3. Construct validity was evaluated through confirmatory factor analysis (CFA), with model fit assessed using CFI, TLI, RMSEA, and SRMR, and compared against established benchmarks (Henseler et al., 2025). Convergent validity was assessed using average variance extracted ($AVE > 0.50$) and composite reliability (CR). Discriminant validity was evaluated via the Fornell-Larcker criterion (Gaskin et al., 2025) and heterotrait-monotrait (HTMT) ratios (Henseler et al., 2025). Structural paths were estimated using maximum likelihood with bootstrapping (5,000 resamples) to test direct and indirect effects. Missing data, comprising less than 5% of the sample, were handled via full information maximum likelihood (FIML). Qualitative data were analysed thematically using deductive codes derived from the theoretical framework and inductive codes emerging from the data.

4. RESULTS

4.1. Error profile and descriptive statistics

Analysis of 381 learner scripts identified 4,072 coded errors across four categories. Table 1 presents the distribution. Conceptual errors dominated overwhelmingly at 65.70%, confirming that deficits in relational understanding, not procedural execution, are the primary driver of poor performance in 3D trigonometry. Transformation/representation errors constituted 18.23%, reflecting persistent difficulty in projecting three-dimensional spatial configurations onto two-dimensional diagrams. Procedural errors (10.84%) and encoding errors (5.23%) were substantially less prevalent.

Table 1. Distribution of error types in learner scripts ($N = 381$).

Error Type	Frequency (%)	Example Manifestation
Conceptual	65.70	Misapplication of trigonometric ratios in 3D solids
Transformation/Representation	18.23	Incorrect projection of 3D figures onto 2D planes
Procedural	10.84	Arithmetic errors in trigonometric calculations
Encoding	5.23	Misreading of angle labels in diagrams

Source: Author

Learners committing primarily conceptual errors scored significantly lower on achievement tests ($M = 42.31$, $SD = 11.24$) than those with predominantly procedural errors ($M = 55.67$, $SD = 9.87$), $t(379) = 8.92$, $p < .001$, $d = 1.23$. This large effect size ($d = 1.23$) underscores the functional severity of conceptual misunderstanding as a performance barrier. Cross-tabulation further revealed a significant negative correlation between CU scores and conceptual errors ($r = -0.45$, $p < .001$) and transformation errors ($r = -0.31$, $p < .001$), while the correlation with procedural errors was non-significant ($r = -0.09$, $p = .12$), suggesting that calculation mistakes operate through a distinct cognitive pathway.

4.2. Measurement model

The CFA confirmed excellent fit: $\chi^2(248) = 512.36$, $p < .001$; CFI = 0.95; TLI = 0.94; RMSEA = 0.06, 90% CI [0.05, 0.07]; SRMR = 0.05. All standardised factor loadings were significant ($p < .001$) and ranged from 0.61 to 0.85, indicating that each indicator contributed substantively to its latent construct. Cronbach's alpha values ranged from 0.84 to 0.90, and composite reliability from 0.78 to 0.88, exceeding established thresholds (Haynes-Brown, 2025). AVE values (0.52-0.64) exceeded the 0.50 benchmark, confirming convergent validity (Gaskin et al., 2025). The Fornell-Larcker criterion was satisfied, as the square roots of AVE (0.72 to 0.80) exceeded all inter-construct correlations (maximum $r = 0.68$), and HTMT ratios remained below 0.85, confirming discriminant validity (Henseler et al., 2015). Measurement invariance across gender groups was established at the scalar level ($\Delta CFI < 0.01$), permitting meaningful group comparisons (Gaskin et al., 2025).

4.3. Structural model and mediation

The structural model demonstrated acceptable fit: $\chi^2(312) = 642.18$, $p < .001$; CFI = 0.95; TLI = 0.94; RMSEA = 0.06, 90% CI [0.05, 0.07]; SRMR = 0.06. All hypothesised direct paths were statistically significant. BL positively predicted CU ($\beta = 0.32$, SE = 0.07, $p < .01$; H₁ supported) and ACH ($\beta = 0.25$, SE = 0.09, $p < .05$), and negatively predicted ME ($\beta = -0.21$, SE = 0.08, $p < .05$; H₁ supported). CU exerted a strong negative effect on ME ($\beta = -0.40$, SE = 0.06, $p < .001$; H₂ supported) and a positive effect on ACH ($\beta = 0.35$, SE = 0.08, $p < .001$). ME negatively predicted ACH ($\beta = -0.28$, SE = 0.07, $p < .01$).

Bootstrapped mediation analysis revealed three significant indirect pathways (H₃ supported). BL → CU → ACH yielded $\beta = 0.11$, 95% CI [0.05, 0.18], $p < .01$. BL → ME → ACH yielded $\beta = 0.06$, 95% CI [0.02, 0.12], $p < .05$. The serial mediation path, BL → CU → ME → ACH, yielded $\beta = 0.10$, 95% CI [0.04, 0.17], $p < .01$, accounting for 28.57% of BL's total effect on ACH. Across all pathways, 64.29% of BL's total effect on achievement operated through the two mediators, indicating that the dominant influence of blended learning on performance is cognitive and indirect rather than direct. The structural model accounted for substantial variance in CU ($R^2 = 0.28$), ME ($R^2 = 0.36$), and ACH ($R^2 = 0.45$), representing moderate to large effect sizes in educational research (Grissom & Kim, 2005). Multi-group analyses confirmed structural invariance across gender ($\Delta\text{CFI} < 0.01$), and the inclusion of prior mathematics achievement as a covariate did not substantively alter path coefficients, confirming robustness.

4.4. Qualitative triangulation

Three themes emerged from interview and observation data, each corresponding to a structural pathway. First, visual-spatial bridging: teachers reported that GeoGebra animations enabled learners to dynamically manipulate angle relationships that static diagrams obscured. One teacher noted that learners who had persistently failed to distinguish angles between lines and planes could suddenly do so after manipulating digital 3D models, a qualitative mechanism explaining the BL → CU pathway (Juandi et al., 2025). Second, cognitive conflict induction: learners reported that peer explanation tasks within digital whiteboard activities exposed contradictions in their reasoning. Articulating a solution to a peer frequently prompted self-correction, providing a classroom-level account of the CU → ME pathway (Baidoo & Luneta, 2024). Third, formative feedback immediacy: learners valued e-quiz feedback that identified and explained errors in real time rather than at the point of summative assessment (Bayaga, 2024). This mechanism directly explains the BL → ME pathway and the overall serial mediation chain.

5. Discussion

The study yields three principal findings, each with theoretical and practical significance. First, BL exerts meaningful effects on achievement primarily through cognitive mediators rather than directly, with 64.29% of its total effect operating indirectly. This undermines naïve assumptions that technology integration improves outcomes through simple exposure; it establishes instead that the quality of pedagogical design determines whether blended environments produce the conceptual restructuring necessary for achievement gains (Dahl & Morch, 2025). Second, the serial mediation path (BL → CU → ME → ACH) advances existing error analysis frameworks (Baidoo & Luneta, 2024) by quantifying how instructional quality translates into misconception reduction through the intermediate development of conceptual understanding. Third, the dominance of conceptual errors at 65.70% of all coded errors, coupled with a large between-group effect size ($d = 1.23$), confirms that practitioner focus on procedural drilling in 3D trigonometry is misallocated; relational understanding is the true performance bottleneck (Skemp, 1978).

The findings carry direct implications for instructional design. Dynamic visualisation tools such as GeoGebra should not be used as digital analogues of static diagrams but as interactive scaffolds that require learner manipulation, as this activates the spatial reasoning processes that underlie CU development (Hamzic & Takaci, 2017). Dialogic tasks, in which learners must articulate and defend solution strategies to peers, are particularly effective at inducing the cognitive conflict necessary for misconception correction (Baidoo & Luneta, 2024). Formative e-quizzes should provide explanation-rich, immediate feedback rather than merely signalling right or wrong, as the qualitative data confirm that feedback quality is a primary driver of error reduction (Juandi et al., 2025).

Several limitations constrain the scope of these conclusions. The study was conducted within a single district, restricting generalisability. The cross-sectional design prevents causal inference about the longitudinal persistence of misconception reduction. BL measurement, while psychometrically robust, relied partly on teacher self-reports, which may introduce social desirability bias. Learners with very low baseline spatial ability appeared to experience limited transfer from digital manipulative tasks to paper assessments, a moderating effect that the SEM model does not capture and that future research should explicitly test.

6. Conclusion

This study provides rigorous empirical evidence that blended learning quality improves achievement in 3D trigonometry primarily by developing conceptual understanding and reducing misconceptions, not through direct instructional transmission. The serial mediation pathway, BL → CU → ME → ACH, offers a parsimonious yet explanatorily rich model of how well-designed blended pedagogy operates cognitively. Practically, this means that investing in instructional design quality, specifically dynamic visualisation, dialogic tasks, and formative feedback, yields greater achievement returns than technology access alone. For policymakers in resource-constrained contexts, the study demonstrates that freely available tools such as GeoGebra, when deployed within a theoretically grounded blended architecture, can substantively disrupt persistent misconceptions that undermine STEM participation. Future research should pursue longitudinal designs, experimental manipulation of specific blended components, and investigation of spatial ability as a moderator of these effects.

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